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LOW THRUST OPTIMAL ORBITAL TRANSFERS
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ABSTRACT

For many optimal transfer problems it is reasonable to expect that the minimum time solution is also the minimum fuel solution. However, if one allows the propulsion system to be turned off and back on, it is clear that these two solutions may differ. In general, high thrust transfers resemble the well known impulsive transfers where the burn arcs are of very short duration. The low and medium thrust transfers differ in that their thrust acceleration levels yield longer burn arcs and thus will require more revolutions. In this research, we considered two approaches for solving this problem; a powered flight guidance algorithm previously developed for higher thrust transfers was modified and an "averaging technique" was investigated.

INTRODUCTION

The problem considered in this research is the optimal transfer of a space vehicle from a circular low Earth orbit (LEO) to a circular geosynchronous (GEO) orbit using a low thrust propulsion system. Because of the advantages of using a low thrust propulsion system for lunar operations, low thrust transfer was researched extensively in the 1960's. Most of the early work done in this area was analytical or crudely approximate; the results obtained for the LEO-to-GEO transfer were for sub-optimal trajectories because of the simplifying assumptions and approximations made to reduce the computations. The flexibility of the algorithms were thus limited and more preflight analysis was required. Thus, it remained to find an efficient numerical scheme to compute an optimal low thrust trajectory using a low thrust propulsion system.

One of the most recent papers which addresses this problem computationally is that of Redding and Breakwell [1]. They compute the gravity losses for a fixed acceleration maneuver as a way of measuring the "transfer efficiency"; the variance in the applied delta v is compared with the impulsive solution. Several authors have studied low thrust trajectories using an averaging approach. Jasper [2] and Sackett et. al. [3] used averaging techniques for the low thrust trajectory minimum-time problem. Sackett et. al. [3] attempted to generalize their results to include the minimum fuel problem.

As with previous research, we expect the results to be beneficial to space exploration. The benefits of low thrust transfers to NASA include: (i) an efficient guidance scheme for orbital maneuvering vehicles, (ii) savings of fuel for the proposed long duration missions, (iii) enable the operation of large fragile space structures, by keeping small the forces they experience, and (iv) the reduction in size of the transfer propulsion system.

OBJECTIVE

The objective of this research was to address the problem of orbital transfers for low thrust propulsion systems. The approach taken was to modify an existing algorithm which was known to work for "higher" thrust propulsion systems and apply it to the low thrust problem; at some lower thrust level, we expected this algorithm would not be effective, and then an averaging technique would be applied. In addition to the fact that the "averaging" approach (as outlined in Appendix 1) was not converging and the previous known algorithm was proving effective for lower thrust, the method of approach was modified. The original powered flight guidance algorithm, OPGUID, was developed in the 1960's and later extended to the multiburn algorithm, SWITCH. (All extensions of the original algorithm are referred to as OPGUID in this report). It is expected that OPGUID can be used as an on-board guidance scheme for future missions (6). The algorithm was known to be effective for intermediate to high thrust maneuvers, and with modifications, it has proven effective for low thrust transfers. The brief formulation described below is taken from that of the authors of OPGUID, Johnson and Brown [4].

Formulation of OPGUID

The equations of motion for a space vehicle are given by

$$\begin{aligned} \dot{\mathbf{r}} &= \mathbf{v} \\ \dot{\mathbf{v}} &= -\frac{\mu\mathbf{r}}{r^3} - \frac{c\dot{m}}{m}\frac{\mathbf{u}}{u} \end{aligned} \tag{1}$$

where $\mathbf{r}$ is the position vector, $\mathbf{v}$ is the velocity vector, the unit vector $\frac{\mathbf{u}}{u}$ is the control vector and $\dot{m}$ satisfying $\dot{m}_{max} \leq \dot{m} \leq 0$, is the vehicle mass rate of change (which represents the magnitude of thrust, and thus is part of control).

A performance index appropriate for minimizing fuel usage is

$$J = \int_{t_0}^{t_f} -\dot{m}\, dt\,. \tag{2}$$

Letting $\mathbf{x} = (\mathbf{r}, \mathbf{v}, m)$ be the state vector and $\mathbf{p} = (\mathbf{q}, \mathbf{s}, w)$ the costate vector, the Hamiltonian is

$$H = L + \mathbf{p}^T\dot{\mathbf{x}} = -\dot{m} + \mathbf{q}^T\mathbf{v} + \mathbf{s}^T\left(-\frac{\mu\mathbf{r}}{r^3} - \frac{c\dot{m}}{m}\frac{\mathbf{u}}{u}\right) + w\dot{m}.$$

According to Pontryagin's Minimum Principle, the optimal thrust direction (the control vector) is

$$\frac{\mathbf{u}}{u} = -\frac{\mathbf{s}}{s},$$

which minimizes the Hamiltonian H. That is,

$$min_{\underset{\sim}{u}}\ H = \left(-1 + w + \frac{sc}{m}\right)\dot{m} + \mathbf{q}^T\mathbf{v} - \frac{\mu}{r^3}\left(\mathbf{s}^T\mathbf{r}\right).$$

Letting S denote the "switching function", which we define as $\left(1 - w - \frac{cs}{m}\right)$, we see that the Hamiltonian is minimized with respect to the thrust direction if $\dot{m} = \alpha$ for $S \leq 0$, and $\dot{m} = 0$ for $S > 0$. It follows by definition that the costate equations are given by

$$\begin{aligned}\dot{\mathbf{q}}^T &= \frac{-3\mu\mathbf{r}^T\mathbf{s}}{r^5}\mathbf{r}^T + \frac{\mu\mathbf{s}^T}{r^3}\\ \dot{\mathbf{s}}^T &= -\mathbf{q}^T\\ \dot{w} &= -s\frac{c\dot{m}}{m^2}.\end{aligned}$$

For high thrust multiburn optimization, the following assumptions are made:
(i) Apart from thrust acceleration, motion is Keplerian.
(ii) Thrust is proportional to mass rate; hence mass loss is zero when thrust acceleration is zero.
(iii) No terminal constraint is time-dependent.
(iv) The number of separate burns are limited to k in order to obtain a realistic optimal solution; otherwise with no penalty, one could insert as many separate burns or coasts as desired.

Thus the boundary value problem requires that trajectory which achieves the desired orbit with minimum fuel expenditure subject to a limit on the total number of separate burn arcs. The dynamical necessary conditions for this boundary value problem are given by (1), (4) and (6). Meanwhile, the boundary conditions for this problem are given at the left end by the initial position, velocity, and mass of the vehicle and at the right end by six conditions defining the desired characteristics of the destination orbit. One of the intermediate point constraints is that the switching function S be zero at each interior switching time. We require that S > 0 on coast arcs and S < 0 on burn arcs. As an illustration, we will consider a circular-to-circular coplanar transfer. Therefore, the initial arc must be a burn arc, i.e. the switching function S is negative and the propulsion system is on. We can not start a circular transfer with a coast because any initial coast time is equivalent to another and thus no optimization of fuel occurs.

Let t_0 be the initial time for which initial position, velocity and mass are given, $\mathbf{r}(t_0) = \mathbf{r}_0$, $\mathbf{v}(t_0) = \mathbf{v}_0$, $m(t_0) = m_0$. Also, let n be the number of separate burn arcs. For the initial burn arc, the switching time for cutoff must be optimized. Clearly, the switching function S must be zero at all switching times. Along the optimal trajectory, the optimality condition on H^*,

$$H^* = S\alpha U(-S) + \mathbf{q}^T\mathbf{v} - \frac{\mu\mathbf{s}^T\mathbf{r}}{r^3} \tag{7}$$

implies that H is identically a constant (does not depend explicitly on t). Suppose that at each switching time t_i, we denote a transversality variable

$$T\mathbf{v} \equiv \mathbf{q}^T\mathbf{v} - \frac{\mu\mathbf{s}^T\mathbf{v}}{r^3}. \tag{8}$$

Since H is constant and the switching function S is zero at each switching time t_i, i = 1, ..., n-1, we obtain the following 2n-2 conditions

$$\begin{aligned} T\mathbf{v}(t_{2j}) &= T\mathbf{v}(t_{2j+1}), j = 1....,n-1 \\ s(t_{2j}) &= s(t_{2j-1}), j = 1...n-1. \end{aligned} \tag{9}$$

The advantage of the last equation is that it allows us to remove the costate variable w from the computations. Clearly, we could not merely let $H(t_i) = H(t_{i+1})$ at each switching time since along coast arcs, we gain no information as far as optimality is concerned.

Thus the boundary value problem is to find the values for the six components of initial costate and the 2n-1 switching times $t_1 ... t_{2n-1}$ such that the results of integrating (1) and (6) forward in time satisfies the six

right end mission conditions at the final cutoff time, the 2n-2 intermediate necessary conditions in (9) and the condition $|u_0| = 1$.
At the final time t_f, k ($\leq$ 6) terminal state constraints are imposed. If fewer than 6 conditions define the destination orbit, then supplementary transversality conditions requiring that the unconstrained orbit parameters be chosen optimally are included to make a total of 6 independent conditions. The most fully defined orbit transfer mission is the 5 orbital constant mission. It is possible, however, to constrain less than 5 orbital constants. Various orbital missions are defined and given by different modes; e.g. mode = 2 constrains the semi-major axis and eccentricity (a and e) of the target orbit. Whereas, mode=5 constrains a, e, and inclination (i), argument of perigee (ω) and right ascension (Ω). For the case of mode 5, if the k constraint functions of final state are given by $g_i(x_f) = 0, i = 1...5$, the conditions of optimality requires that the final costate p_f lie in the space spanned by the gradients of the constrained functions. Thus is a_i span the space orthogonal to the space spanned by the gradients $\frac{\partial g_i}{\partial x}$ of g_i, then p_f must be orthogonal to the single vector a_6. We can see that this transversality vector must be orthogonal to the gradient of every final constraint, and thus we can take

$$a_6 = \left(\mathbf{v}, -\frac{\mu \mathbf{r}}{r^3}\right)^T.$$

MODJFICATION FOR LOW THRUST ACCELERATION

The long length of the burn arcs for lower thrust acceleration can introduce great sensitivity in an orbital transfer, especially in the initial guesses for costate variables. Because of this sensitivity in the many costate variables, as well as the initial guesses of the coast and burn times, the multiburn case for low thrust acceleration has proven difficult to converge. The first attempts in applying this algorithm for transfers shorter than LEO-to-GEO, using multiple burn-coast arcs did not converge; and in several cases, the converged solution was exactly the same as the single burn arc solution depending upon the initial array of times.

Example.

The savings in fuel and burntime can be seen in the following example, where we have convergence. We compute the circular orbital transfer from a = 6656 km to 6756 km using MODE 2 of OPGUID. Thrust = 2.646 kn, initial mass = 270000 kg, mass rate = .60, and isp = 450 were the vehicle capability parameters used.

For the single burn case, an initial cutoff time of 20,000 seconds was used to obtain the following results within 26 iterations using the velocity vector direction in the initial guess of costate: semimajor axis a = 6755.9, eccentricity = .00050, total burn time of 18,948.8 seconds = 5.26 hours, final mass = 258630.70 kg.

For the two burn case, the initial times array representing a 12,000s burn-5000s coast-5000s burn, with the same initial guess of costate above, converged within 85 iterations to the orbit with semimajor axis a = 6755.6, eccentricity = .0010, total burn time of 17,089.9 seconds = 4.75 hours, final mass = 259746.07 kg. The final times array is: 0, 0, 0, 13771.50, 17462.90, 20781.28.

Since we expect long burn arcs, it is not reasonable to expect that a single burn LEO-to-GEO transfer is the "best" approach to fuel efficiency, so we take advantage of the success we found in applying OPGUID to "short" burns in the single burn case. We used the following approach:

A circular transfer LEO —GEO transfer is obtained by:

(i) optimizing a burn of specified length at perigee

(ii) coast around the orbit and center the next burn about perigee

(iii) raise apogee to desired semimajor axis by successive burns centered at perigee

(iv) coast to apogee and optimize a burn of specified length at apogee

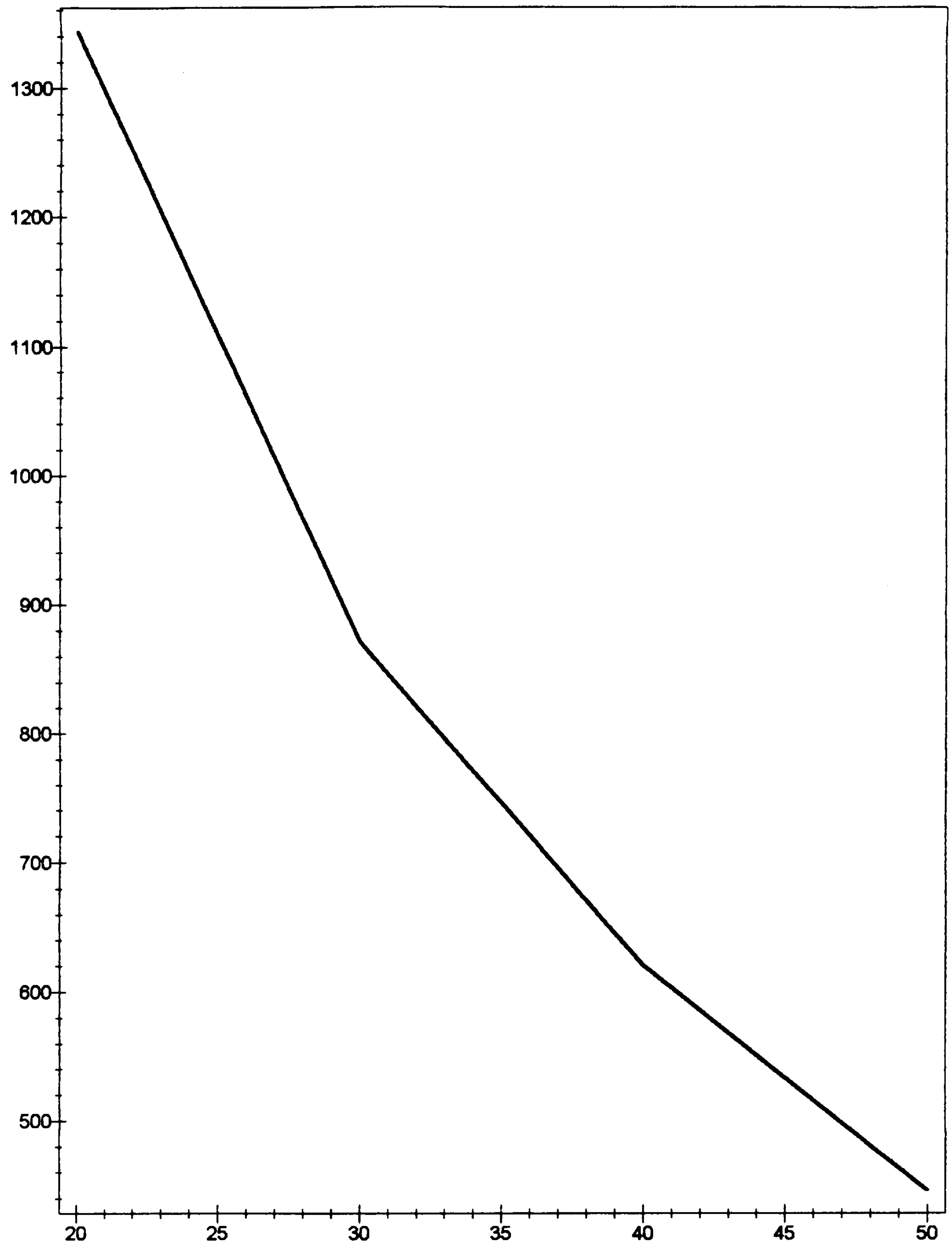
(v) raise perigee by successive burns centered about apogee in order to circularize orbit.

The results obtained for MODE = 2, where the semimajor axis and eccentricity are constrained is given in the table and graphs below. As expected, there is a trade off between time of transfer and savings in fuel.

LEO-TO-GEO CIRCULAR TRANSFERS
USING VARIOUS BURN LENGTHS

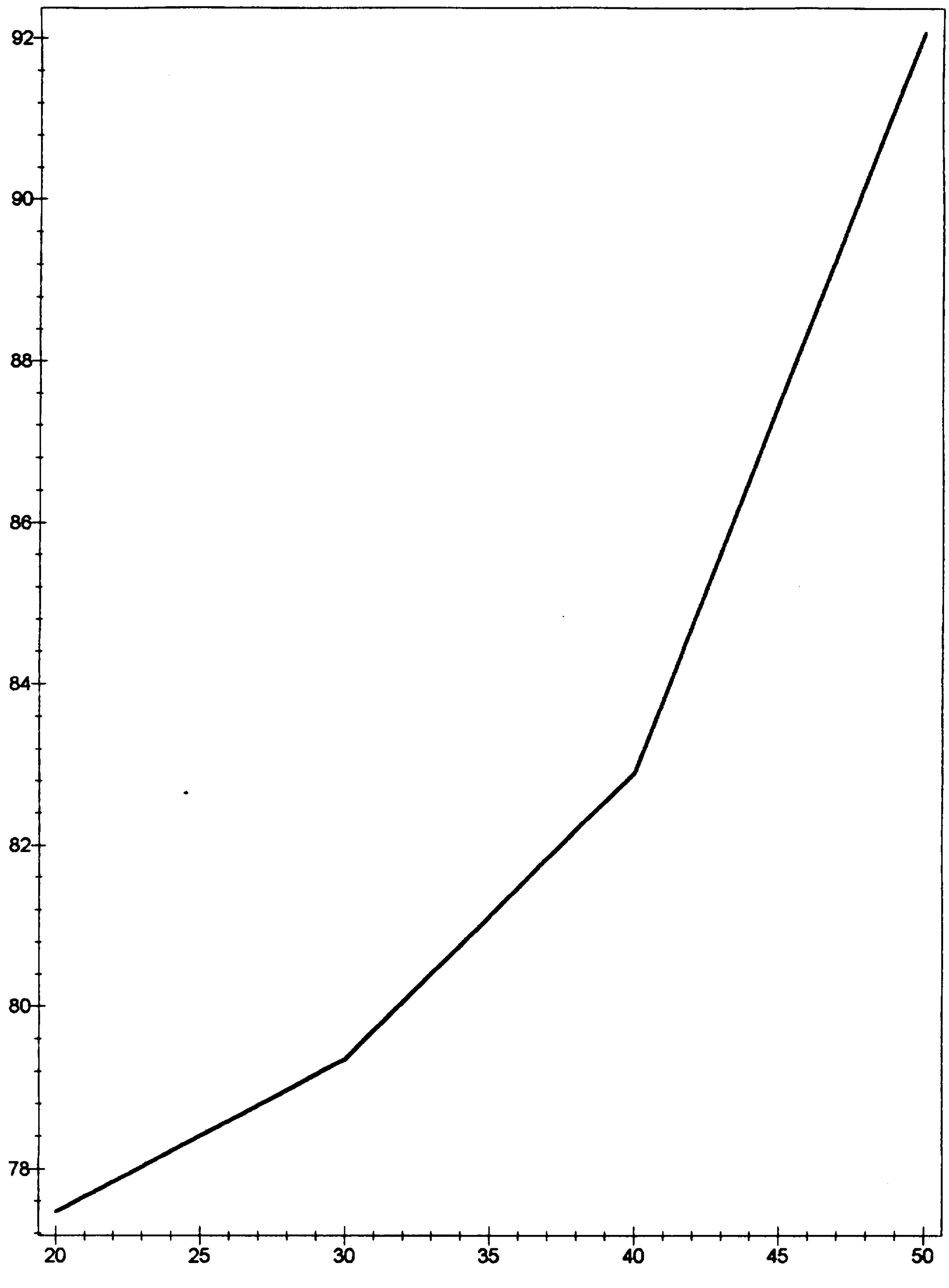
Burn length (minutes)		20	30	40	50
Number of perigee burns		152	101	75	64
Numer of apogee burns		59	38	27	20
Burntime (hours)		77.47	79.35	82.90	92.07
Totaltime (hours)		1344.04	871.33	620.65	446.51
Final Mass (kg)		102,655	98,163	90,946	71,641

Burnlength versus Total Time



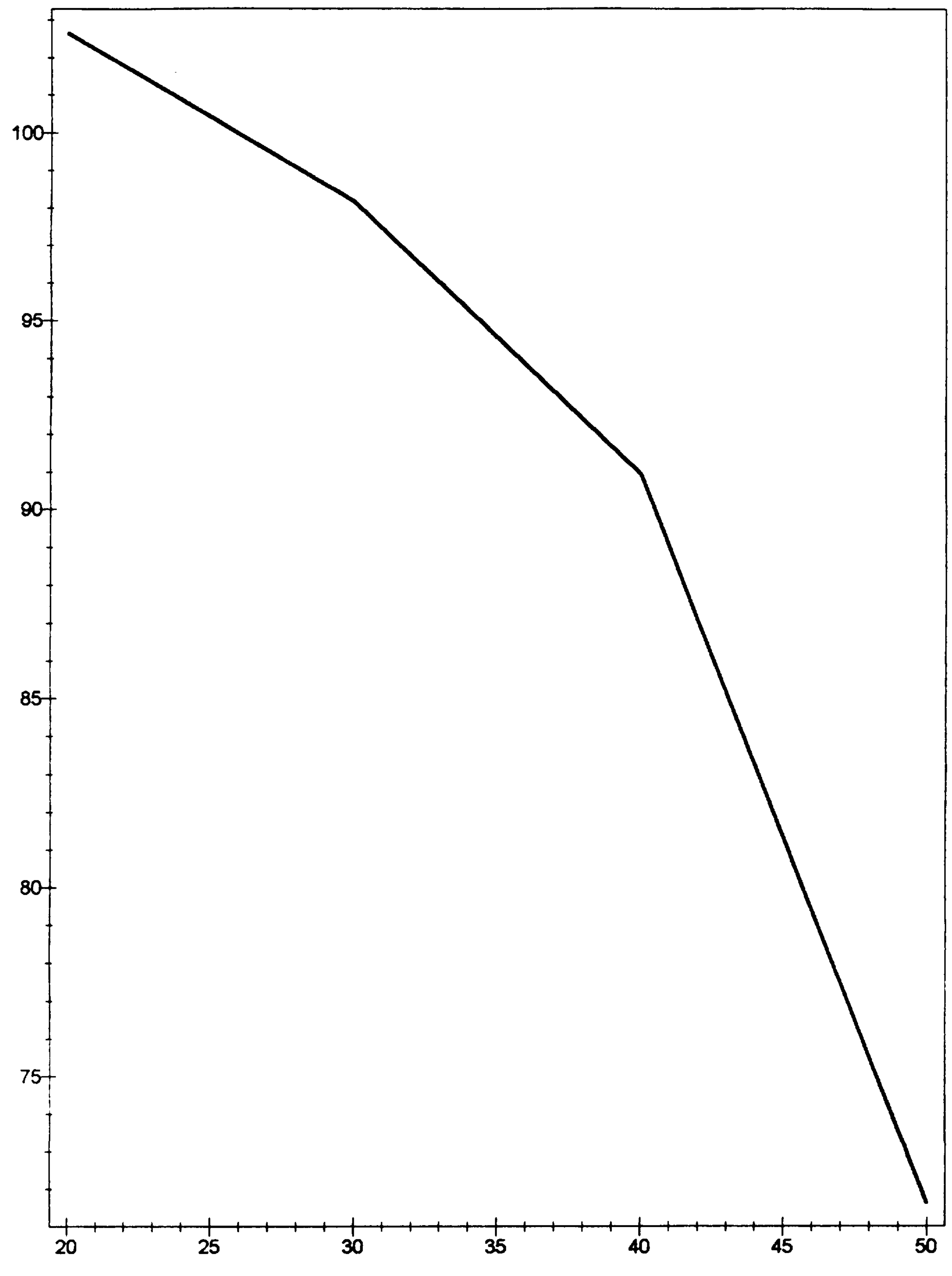
10A

Burnlength versus Total Burntime



10B

Burnlength versus Final Mass times .001



10C

CONCLUSIONS

It is apparent that the problem of finding the initial costate is the most difficult aspect of computing optimal low thrust orbital transfers, whether one uses OPGUID or MINFUEL. In both cases, for circular-to-circular transfers, taking the initial thrust direction (which is aligned with the initial costate vector) in the direction of the velocity vector has proven the most effective method of finding a solution. One of the disadvantages of using OPGUID is that one has to guess the final cutoff time (and intermediate switch times for multiburn cases). In addition to using the impulsive transfer to guess the final cutoff time, we also looked at other algorithms, such as SCOOT (Simplex Computation Of Orbital Transfers) to guess the length of burn arcs.

We were able to use OPGUID effectively to compute the low thrust transfer from LEO-TO-GEO using successive single burn transfers; however, we were unable to obtain convergence for a similar multiburn approach. One would expect that there would be more savings in fuel if we added a coast arc to the transfer; this is supported in the above example. For the circular transfer case, a possible multiburn approach would be to circularize at various radii as we approach geosynchronous orbit.

CALLING PROGRAM CALLTOGUIDE

Purpose: This is the calling program to the modules which computes the minimum fuel transfer. CALLTOGUIDE actually makes a call to the subroutine G715P_G which then makes the call to GUIDE which computes the trajectory across burn and coast arcs. CALLTOGUIDE reads in the necessary data for the transfer, such as vehicle capabilities and initial and target orbits. Then G715P_G is called to make successive burns centered around a "fixed" perigee and then to coast around in order to center the next burn about perigee; when the desired apogee is reached, a coast is made to center the next burn around this apogee and successive burns and coasts are performed until the desired perigee has been reached. In the cases of circular target orbits, we circularize at this apogee. Also, note that the initial data is given in terms of orbital elements and is then changed to a state vector using the subroutine KEPSTATE.

INPUT VARIABLES:

THLIQ—thrust level in kilonewtons

NCOAST—the number of coast arcs per leg of transfer

NBURN—the number of burn arcs per leg of transfer

TMODE—the mode of transfer; the desired constraints on the target orbit determines the mode; e.g.TMODE=2 constrains the semimajor axis and the eccentricity of the final orbit; same as GUID_OPTION

ITARG—the value of 2 tells G715P_G to compute a Cartesian target vector

DINCL, DNODE, THT, ARGPER—target values of inclination, ascending node, flight path angle and argument of perigee

IFLAG—the value of 0 implies no initial coast arc

VEH(I,J)—Ith leg, Jth vehicle characteristic in that leg
- J=1 — initial mass of leg, kg
- J=2 — fuel flow rate, kg/s
- J=3 — total burn time of leg, s (set to a very large number)
- J=4 — thrust magnitude, kilonewtons
- J=5 — 0
- J=6 — 0
- J=7 — initial Runge-Kutta integration step size for ith leg, s

TIMES(1:6) — array of engine on/off times, s

ICIRC — the value of 1 indicates a noncircular target orbit; while 0 indicates a circular target orbit

DELTABURN — the length of burn arcs to be optimized, s

RPER, RAPO — the radius of perigee and apogee of initial orbit, km

CEXV — the calculated exhaust velocity

SMATARG — the target semimajor axis, km

Note that there are some variables in the input list which are either constant or are set to certain values for convenience in the computation of an orbital transfer.

MAIN VARIABLES:

DELTAM — the change in mass for a burn of length deltaburn

DELTAV — the velocity increment caused by a burn of length deltaburn

R0, V0 — the current radius and velocity for each intermediate transfer

RT, VT — the target radius and velocity for each intermediate transfer

PERRAD, APORAD — the current radius of perigee and apogee

TIMEPER — the time elapsed since perigee passing of vehicle

APOTIME — the necessary time to reach apogee of current orbit from current position

COASTTIME — the calculated length of coast arcs needed to center burn about perigee and apogee

BURNTIME — the length of actual burn arc after optimizing the burn length deltaburn

TOTALTIME — total time for transfer, which is the sum of each intermediate transfer

OP_GUID_DATA_INIT

Purpose: This routine initializes data needed in the integration process and the earth data.

G715P_G

Purpose: This subroutine has been substantially changed from the original module of Dukeman which was used as a calling program with many mission components, including ascent and descent, engine throttling and so forth. Although the comments have been left unchanged, the purpose is to set up the target orbit in Cartesian coordinates, set up the initial costate and compute yaw and pitch angles, if desired, and of course to call the subroutine GUIDE to compute the trajectory over burn and coast arcs.

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APPENDIX 1

AVERAGING APPROACH

The formulation of the problem follows that of Horsewood et. al. [7], where the state of the spacecraft is given in terms of the slowly varying equinoctial elements, which are expressed in terms of the classical elements a, e, i, ω, Ω as

$$\mathbf{z} = (a, e\sin(\omega+\Omega), e\cos(\omega+\Omega), \tan(i/2)\sin\Omega, \tan(i/2)\cos\Omega)^T.$$

The spacecraft mass, m, is also a state variable. In addition, the position of the spacecraft within an orbit is given by the eccentric longitude, $F = E + \omega + \Omega$.

The equations of motion are

$$\dot{\mathbf{z}} = \frac{2P}{mc} M\hat{\mathbf{u}}, \dot{m} = -\frac{2P}{c^2} \tag{1}$$

where $M = \frac{\partial \mathbf{z}}{\partial \mathbf{r}}$ is a 5×3 matrix calculated by treating F as an independent parameter, such that the variation of F with respect to the other state variables is zero; P is the power due to the thrusters, c is the exhaust velocity and $\hat{\mathbf{u}}$ is the unit vector in the direction of thrust.

It follows from the well-known maximum principle that the fuel optimal trajectory from a given state to some desired final state is found by thrusting at every point along the trajectory in the direction which maximizes the Hamiltonian function H. H can be written in terms of the state variables and their corresponding costate (adjoint) variables $\lambda_{\mathbf{z}}$ and λ_m as

$$\begin{aligned} H &= \lambda_{\mathbf{z}}{}^T \dot{\mathbf{z}} + \lambda_m \dot{m} \\ H &= \frac{2P}{mc}\left(\lambda_{\mathbf{z}}{}^T M\hat{\mathbf{u}} - \frac{m\lambda_m}{c}\right) \end{aligned} \tag{2}$$

where the costate variables satisfy a first order linear system of ordinary differential equations. Now, to maximize H we need only thrust in the direction given by $\hat{\mathbf{u}} = \frac{M^T\lambda_{\mathbf{z}}}{|M^T\lambda_{\mathbf{z}}|}$. Note that if the quantity in parenthesis in [2], call it σ, is negative, then H is negative and hence H is maximized by letting P = 0, which amounts to turning the propulsion system off , i.e. "coasting"; on the other hand, if σ is positive, then H is maximized by letting P take on its maximum value, i.e. "thrusting".

Because of the many orbit revolutions of a long duration transfer, the intensive computations can be reduced by an averaging technique. We can compute an "averaged" Hamiltonian function by holding the state and costate variables constant over an orbital period of duration τ, i.e. we assume Keplerian motion, and integrating the actual Hamiltonian function as follows:

$$\tilde{H} = \frac{1}{\tau}\int_0^{\tau} H(\bar{\mathbf{z}}, \bar{\lambda}_{\mathbf{z}}, \bar{m}, \bar{\lambda}_m, F)dt$$

$$= \int_0^{2\pi} H(\bar{\mathbf{z}}, \bar{\lambda}_{\mathbf{z}}, \bar{m}, \bar{\lambda}_m, F)s(\bar{\mathbf{z}}, F)dF$$

where $s(\bar{\mathbf{z}}, F) = \frac{1}{\tau}\frac{dt}{dF}$. The "averaged" equations of motion can now be computed using this "averaged" Hamiltonian, and since the Hamiltonian and its derivatives are zero during coast phases, we need only integrate these equations over the predetermined thrust intervals.

NUMERICAL APPROACH

A FORTRAN program MINFUEL has been written using the averaging approach as outlined above for the minimum propellant low thrust circular transfer. The initial orbit: semimajor axis a = 6656 km, eccentricity e = 0, inclination i = 10 deg, ascending node Ω = 0 deg, argument of perigee ω = 0 deg, and eccentric longitude F = 0 deg. The initial mass m = 270000 kg, exhaust velocity cexv = 4.41 km/s, mass rate mrate = —.6 kg/s and thrust T = 5.843443 kn. The final state desired is a = 6756 km, e = 0, and the final transversality condition desired is costate mass = 1. The algorithm of MINFUEL is given below.

(1) In order to obtain the desired final conditions, we must iterate on the unknown initial costate. An initial costate guess can be found by using the fact that it is fuel-optimal to thrust in the direction of motion, or tangential direction. This known direction can be used to help guess the initial costate values since $\hat{\mathbf{u}} = \frac{M^T\lambda_{\mathbf{z}}}{|M^T\lambda_{\mathbf{z}}|}$. Given any state, this optimal thrust direction can be calculated in terms of the costate values by running the program XFORM to compute the transpose of the matrix M and the velocity vector. These values were scaled upon input to MINFUEL. Since we know that it is optimal to start a circular transfer with a thrust arc, the initial value of costate mass was found using

the scaled values of costate to ensure that the switch function was a "small" positive number. Module used is SWITCH2.

(2) The iterator is called to begin finding the desired initial costate; The SECANT algorithm is the one currently encoded, with a call to the singular value decomposition subroutines, SVDCMP and SVBKSB, to compute the NEWTON corrections for the calculated Jacobian matrix.

(3) A trajectory corresponding to each of the initial costate guesses is found by calling the subroutine FUNCT, which in turn calls the Runge-Kutta routine to integrate the averaged equations of motion over a time step. While performing the integration, a quadrature routine is called to average the state, costate and Hamiltonian function, and hence the averaged equations of motion; it is essential that first the switch points, the zeroes of the switch function, around the orbit be determined a priori in order that this integration is valid. The switch function can be expressed in terms of the fast variable F around an orbit and the zeroes of the switch function $\sigma = \left[\lambda_z^T M M^T \lambda_z\right]^{\frac{1}{2}} - \frac{m\lambda_m}{c}$ are found, which defines the thrusting subintervals of $[0, 2\pi]$ for integration. Again for the circle-to-circle transfer we know that initially the switch function should be positive.

(4) The final conditions are evaluated to determine if they have been satisfied. The Newton corrections are then computed using the Jacobian matrix. We continue until final conditions are within desired tolerance.

Test Results

The result of running MINFUEL is that it does not converge to the desired final state and costate mass. Initially, it appears to converge, then it diverges. Whether, this is the initial costate guess, or some element of the code has not been resolved.

CALLING SEQUENCE AND VARIABLES

SUBROUTINE INPUT—Initializes orbits and transfer data
MAIN VARIABLES:
ZCUR — Current state variables of equinoctial elements a,h,k,p,q, and mass m
COCUR — Current costate variables, input variables as Lam(6)

SUBROUTINE ITERN—Calls the numerical iterator for the initial costate (currently SECANT)

SUBROUTINE SECANT—Written by Greg Dukeman of MSFC

SUBROUTINE FUNCT—Calculates the optimal trajectory corresponding to the given initial costate guess; calls the Runge-Kutta routine and evaluates the final conditions

SUBROUTINE SWCHPTS—Compute a priori the zeroes of the switching function which are needed in the averaging process in
QUAD to integrate about the orbit
MAIN VARIABLES:
NROOT — number of roots found
ROOTS — actual zeroes found
DERZ12 — averaged derivatives

SUBROUTINE INTEG—Integrate the averaged equations of motion and Hamiltonian

SIGMAF —The switching function, which is the magnitude of the primer vector

```
      PROGRAM CALLTOGUIDE
C*****************************************************************************
C  CALLTOGUIDE is the calling program to the modules of OPGUIDE to compute the
C  optimal trajectory of transfer by raising apogee using successive perigee
C  burns and then raise perigee in order to attain the desired semi-major axis.
C
C******************************************************************************
C  Many of the main variables are defined in module G715P_G of OPGUID
C       TIMEPER--Time elaspsed since perigee passing
C       APOTIME--Time needed to reach apogee of an orbit
C       COASTTIME--Length of coast arc needed to center burn arcs at perigee and
C                --apogee points
C       DELTABURN--Length of burns to be optimized in transfers
C       TOTALTIME--Total time of burn and coast arc lengths
C******************************************************************************

      IMPLICIT REAL*8 (A-H, O-Z)
      REAL*8 ISP, NMEAN, MANOM
      INTEGER  TGT_SET, GUID_OPTION
      PARAMETER (MAXBURN=600,XMU=3.9860064D+14,PI=3.14159654)

      COMMON /LINEAR/ LINEAR_SOLUTION_METHOD
      COMMON /OPGUID_DATA/ ISP,THLIQ,NCOAST,NBURN,
     $        ALPHA_MIN,ITARG,MITR,G0,CEXV,AZ,PHIL,C3OA,DECOA,RAOA,
     $        GUID_OPTION

      COMMON /GIDIN/ XT(6),TTG,X0(6),T0,XMASSINIT,VEH(10,7),Q0(6),TIMES(6),
     $        CC(6)
      COMMON /ORBIT_INFO/ DINCL,DNODE,THT,ARGPER,TIME,IFLAG,
     $        ICIRC,TGT_SET,DELTABURN,R0,V0,RT,VT,XMASS,TMODE

      COMMON /Q0SUB/ Q0_SUB(6),Q1_SUB(6)
      COMMON /CURRENT_STATE/ XCUR(6)
      COMMON /MAXKOUNT/ ITER
      COMMON /BEST_COSTATE/ X0_INIT(6),Q0_INIT(6),Q0_BURN(6)
      LOGICAL LAST_BURN, PERBURN

C ASSUME THAT SOFT_CONSTRAINTS AND NORM_VARS ARE FALSE
C THE VARIABLES IN THE NAMELIST BELOW ARE DEFINED IN OPGUID

      NAMELIST /G715P_C/ ISP,THLIQ,NCOAST,NBURN,TMODE,GUID_OPTION,
     $        ITARG,MITR,SOFT_CONSTRAINTS,LINEAR_SOLUTION_METHOD,OBLATE,
     $        NORM_VARS,DINCL,DNODE,THT,ARGPER,XMASSINIT,TIME,T0,IFLAG,
     $        VEH,TIMES,ICIRC,TGT_SET,DELTABURN,RPER,RAPO,
     $        G0,CEXV,AZ,PHIL,C3OA,DECOA,RAOA,ALPHA_MIN,Q0_SUB,Q1_SUB,SMATARG

      CALL OP_GUID_DATA_INIT

      READ(100, G715P_C)

      UK = XMU/1.D+9

C     INITIALIZE ORBIT DATA
      TOTALTIME = 0.
      NUM_APO_MAX = 200
      NUM_PER_MAX = 400
      NUM_PER_BURN = 0
      NUM_APO_BURN = 0
      LAST_BURN = .FALSE.
      PERBURN = .TRUE.
      SMA0=(RPER+RAPO)/2.0
      ECC0=1.0 - (RPER/SMA0)
      TRUAN = 0.
      CALL KEPSTATE(X0,X0(4),R0,V0,SMA0,ECC0,DINCL,ARGPER,DNODE,TRUAN)
C     THE ANGLES ARE RETURNED IN RADIANS FROM KEPSTATE
      RADDEG = 180./PI
      DEGRAD = PI/180.

C     NECESSARY DATA FOR G715P_G TO COMPUTE TARGET ORBIT
      XMASS = XMASSINIT
      DELTAM = VEH(1,2)*DELTABURN
      DELTAV = CEXV*(DLOG(XMASS/(XMASS-DELTAM)))
      RT = RPER
      RAPOTARG = SMATARG
      VT = V0 + DELTAV
      CALL G715P_G

      NUM_PER_BURN = NUM_PER_BURN + 1

      DO I = 1, 6
        XCUR(I) = XCUR(I)*1000.
      END DO
      CALL CSVTOORBELE(XCUR,XCUR(4),AC,EC,DINC,OGAC,APEC,FC)
      DO I = 1, 6
        XCUR(I) = XCUR(I)/1000.
        X0(I) = XCUR(I)
      END DO
      WRITE(30,*) ' '
      WRITE(30,*) 'THIS IS THE END OF BURN ',NUM_PER_BURN,' AT PERIGEE.'
      WRITE(30,*) 'CURRENT ELEMENTS: SMA',AC*.001,' ECC ',EC,' INC  ',DINC,
     & ' RAN ',OGAC,' ARGOFP ',APEC,' TAN ',FC,' RAPO ',AC*.001*(1.+EC),
     & ' RPER ',AC*.001*(1.-EC)

      AC = AC/1000.
      FC = FC*DEGRAD
      PERRAD = AC*(1.0 - EC)
      APORAD = AC*(1.0 + EC)
      TOTALTIME = TOTALTIME + TIMES(6)
      BURNTIME = TIMES(6) - TIMES(5)
      TOTAL_BURNTIME = TIMES(6)
      WRITE(30,*) 'NUMBER OF ITERATIONS: ',ITER
      WRITE(30,*) 'TOTALTIME AND BURNTIME ARE: ',TOTAL_BURNTIME

C     FOR NONCIRCULAR TARGETS, THEN TARGET PERIGEE AND APOGEE POINTS ARE GIVEN
C     BELOW FOR A TARGET ECCENTRICITY
      DO I = 1, MAXBURN
      DELTAF = PI - FC
      T0 = 0.
      TIME = 0.
      XMASSINIT = XMASS
      TANE2 = SQRT((1.0-EC)/(1.0+EC))*TAN(FC/2.0)
      EANOM = ATAN(TANE2)*2.0
      MANOM = EANOM - EC*SIN(EANOM)
      NMEAN = SQRT(UK)/AC**(1.5)
      TIMEPER = (MANOM)/NMEAN
      APOTIME = PI/NMEAN - TIMEPER
      PERIODT  = PI*SQRT((AC**3)/UK)
      DELTAM = VEH(1,2)*DELTABURN
      DELTAV = CEXV*(DLOG(XMASS/(XMASS-DELTAM)))

C     AFTER FIRST BURN AT PERIGEE, BEGIN ITERATION PROCESS OF BURNS AROUND
C     PERIGEE AND COASTING TO APOGEE AND PERFORM BURNS ABOUT APOGEE UNTIL
C     DESIRED SEMI-MAJOR AXIS IS REACHED.

      IF (PERBURN) THEN
        NUM_PER_BURN = NUM_PER_BURN + 1
        WRITE(30,*) ' '
        WRITE(30,*) 'AT THE END OF COAST-BURN',NUM_PER_BURN,' AT PERIGEE.'
        COASTTIME = APOTIME + (PERIODT- DELTABURN/2.0)
        V0 = SQRT((UK/AC)*(1.0+EC)/(1.0-EC))
        VT = V0 + DELTAV
        IF (APORAD .GE. (RAPOTARG)) PERBURN = .FALSE.
```

```
            APOGEE_REACHED = APORAD
            IF (NUM_PER_BURN .GT. NUM_PER_MAX) PERBURN =.FALSE.
          ELSE
            NUM_APO_BURN = NUM_APO_BURN + 1
            WRITE(30,*) ' '
            WRITE(30,*) 'AT THE END OF COAST-BURN ',NUM_APO_BURN,' AT APOGEE.'
            RT = APOGEE_REACHED
            IF (AC .GE. (SMATARG)) THEN
C               CIRCULARIZE AT APOGEE
              ICIRC = 0
              RT = SMATARG
              VT = SQRT(UK/SMATARG)
              LAST_BURN = .TRUE.
            ELSE
              V0 = SQRT((UK/AC)*(1.0-EC)/(1.0+EC))
              VT = V0 + DELTAV
            ENDIF
              COASTTIME = APOTIME - (DELTABURN/2.0)
          ENDIF

          IF ((NUM_APO_BURN .EQ. 0) .AND. (.NOT. PERBURN)) THEN
            NUM_PER_BURN = NUM_PER_BURN - 1
            GO TO 15
          ENDIF

          TIMES(5) = COASTTIME
          TIMES(6) = COASTTIME + DELTABURN
          CALL G715P_G
          BURNTIME = TIMES(6) - TIMES(5)
          TOTAL_BURNTIME = TOTAL_BURNTIME + BURNTIME
          TOTALTIME = TOTALTIME + TIMES(6)
          DO J = 1, 6
            XCUR(J)=XCUR(J)*1000.
          END DO
          CALL CSVTOORBELE(XCUR,XCUR(4),AC,EC,DINC,OGAC,APEC,FC)
          DO J = 1, 6
            XCUR(J)=XCUR(J)/1000.
            X0(J)=XCUR(J)
          END DO
          WRITE(30,*) 'CURRENT ELEMENTS:  SMA ',AC*.001,' ECC ',EC,' INC ',
     &    DINC,'RAN ',OGAC,' ARGOFP ',APEC, ' TAN ',FC,' RAPO ',
     &    AC*.001*(1.+EC),' RPER ',AC*.001*(1.-EC)

          WRITE(30,*) 'THE NUMBER OF ITERATIONS WERE: ',ITER
          WRITE(30,*) 'THE TOTAL TIME TAKEN WAS:', TIMES(6)
          WRITE(30,*) 'BUT THE BURNTIME WAS ONLY:', BURNTIME
          WRITE(30,*) ' '

          AC = AC/1000.
          FC = FC*DEGRAD
          PERRAD = AC*(1.0 - EC)
          APORAD = AC*(1.0 + EC)
          IF (LAST_BURN .OR. (NUM_APO_BURN .GE. NUM_APO_MAX)) THEN
            WRITE(30,*) ' '
            WRITE(30,*) 'A FINAL ORBIT HAS BEEN REACHED WITH SEMIMAJOR AXIS'
            WRITE(30,*) 'AND ECCENTRICITY OF ',AC,EC
            WRITE(30,*) 'THE TOTAL TIME FOR TRANSFER WAS',TOTALTIME
            WRITE(30,*) 'THE TOTAL BURNTIME WAS',TOTAL_BURNTIME
            WRITE(30,*) 'THE FINAL MASS IS ',XMASS
            WRITE(30,*) 'THE NUMBER OF PERIGEE BURNS WERE ',NUM_PER_BURN
            WRITE(30,*) 'THE NUMBER OF APOGEE BURNS WERE ',NUM_APO_BURN
            GO TO 10
          END IF

   15     CONTINUE
          END DO
```

```
  10    CONTINUE
  132   FORMAT(6(F12.5,1X))
        END

      SUBROUTINE OP_GUID_DATA_INIT
      IMPLICIT REAL*8(A-H, O-Z)
C  Earth constants are IAU, 1964
      DATA REARTH/6378.160/, OBLATE/.10827E-2/,
     $     OMEGA/0.729211585D-4/, ETA/1.0/ ! Kilo units
      PARAMETER ( PI = 3.141592654D0 )

      COMMON /CLEG/ ATP(7), TL, HMAX, EVT, H0
      COMMON /CPHYS/ UK, REARTH,RHO0,OMEGA,OBLATE, OBLATE_FACTOR
      COMMON / CWT / WT(6)

      WT(1) = 0.95                   !  weights for soft constraints guidance mod
      WT(2) = 0.95
      WT(3) = 0.95
      WT(4) = 0.95
      WT(5) = 0.95
      WT(6) = 0.
      EVT = 1.D-5                    ! desired error level in guidance modelling

      RETURN
      END

      SUBROUTINE G715P_G

C  ********************************************************************
C
C  INPUTS:  X013(1:3) - vehicle inertial position vector, km, AP13 coord.
C           X013(4:6) - vehicle inertial velocity vector, km/s, AP13 coord.
C           AZ        - launch azimuth, rad
C           PHIL      - launch latitude of launch site, rad
C           ARGPER    - target argument of perigee measured from the
C                       descending node positive in direction of flight, rad
C           DINCL     - target inclination, rad
C           DNODE     - target descending node, rad
C           RT        - target radius magnitude, km
C           VT        - target velocity magnitude, km/s
C           THT       - target flight path angle, rad
C           TIMES(1:6) - array of engine on/off times, referenced to
C                        beginning of mission, s
C           VEH(I,J)  - Ith leg, Jth vehicle characteristic in that leg
C                   J=1  - initial mass of leg, kg
C                   J=2  - fuel flow rate during non-g-limited legs, kg/s
C                   J=3  - total burn time of leg, s
C                   J=4  - when no g-limit exists, exhaust velocity
C                                      * abs(fuel rate), kg km/s^2
C                          when g-limit exists, thrust acceleration magnitude,
C                                                                 km/s^2
C                   J=5  - when not g-limited, = 0
C                        - when g-limited, -(g-limit/exhaust velocity), s^-1
C                   J=6  - 0, for exoatmospheric legs
C                        - reference area of vehicle, km^2
C                   J=7  - initial Runge Kutta integration step size for
C                          ith leg, s
C                   DELP - the most recent pitch gimbal angle from routine GIMBAL
C                   DELY - the most recent yaw gimbal angle from routine GIMBAL
C                   EXIT_AREA - total engine exit area, m^2
C                   TCSAVE(1:20) - coast times after each stage, s
C                   ITIME - ?
C                   DTN - ?
C                   E2 - target orbit eccentricity
C                   FM - magnitude of vehicle acceleration excluding gravity, m/s^2
```

```
C                     GT - negative of magnitude of grav acc at target radius, m/s^2
C                     GX - gravity vector in NLS plumbline coord system, m/s^2
C                     IBURNG - thrust stage indicator
C                     DT_GUID - guidance cycle time, s
C                     IOP1 - IGM parameter
C                     NBURNG - total # of thrust stages
C                     NCYL - ?
C                     PHIT - ?
C                     TAU(1:20) - (vehicle mass / mass flow rate), s
C                     TCG(1:20) - coast times after each stage ?, s
C                     TFG(1:20) - time to enforce fixed attitude-rate for each stage,
C                     TGG(1:20) - burn time for each stage, s
C                     TI - ?, s
C                     TPHIT - ?
C                     XPM1, XYM1 - past values of pitch and yaw, rad
C                     XDLIMP, XDLIMY - maximum allowable values for pitch and yaw rat
C                     GLIMIT - g-limit, g's
C                     MCT - ?
C                     IG2(1:20,1:2) - ?
C                     CHIP - ?
C                     CHIY - ?
C                     TIME - time elapsed since launch, s

C
C  OUTPUTS: CHIPCN    - commanded pitch angle, radians
C           CHIYCN    - commanded yaw angle, radians
C           UT0       - unit thrust direction in AP13 coord.
C           UDT0      - time derivative of unit thrust direction in AP13
C                                                           coord., s^-1
C           CHIDP, CHIDY - commanded inertial yaw rate with respect to NLS plumblir
C                                                                       coord sy
C  ***********************************************************************************

      IMPLICIT REAL*8(A-H,O-Z)
      REAL*8 ISP,M0,INCT
      INTEGER TGT_SET, TGT_SET_OLD
      DATA TGT_SET_OLD / 0 /
      PARAMETER(PI=3.141592654D0,XMU=3.9860064D+14)
      LOGICAL SAVED_GUID_PARAMS
      COMMON/GIDIN/XT(6),TTG,X0(6),T0,M0,VEH(10,7),Q0(6),TIMES(6),CC(6)
      COMMON /CAIN/ W(3),CLA0,CA0,ETA,AX(3), RHO_C1, RHO_C2, RHO_C3,
     $              RHO_C4, PRES_C1, PRES_C2, PRES_C3, VSOUND_C1,
     $              VSOUND_C2, VSOUND_C3, VSOUND_C4, CA_C1, CA_C2,
     $              CA_C3, CA_C4, CA_C5, CNA_C1, CNA_C2, CNA_C3,
     $              CNA_C4, CNA_C5
      COMMON /CINDEX/ NARC,IARC,JMAX,JM,JMAX1,JLAST,NO,NOP,NRKGOS
      COMMON /CLEG/ ATP(7), TL, HMAX, EVT, H0
      COMMON /CPHYS/ UK, REARTH,RHO0,OMEGA,OBLATE, OBLATE_FACTOR
      LOGICAL SOFT_CONSTRAINTS
      COMMON /CMODE/ MODE, IFREZ, ISTOP, SOFT_CONSTRAINTS, MEANTGT
      COMMON/GIDOUT/DQ0(6),DTIMES(6),E(12,12),DC(12),XG(12),
     1              Z(12,12),DD(6),SM(5), CK
      COMMON/GUID_PARAMS/SAVED_GUID_PARAMS,VR_0(3),CSV1_0,CSV2_0
      COMMON /TABLE_SIZES/ NP_CFBST,NP_CFBST_GUID,NP_CAT,
     $ NP_CAT_GUID,NP_CHTABL,NP_CLABT,NP_CLABT_GUID,NP_CMAT,
     $ NP_CMAT_GUID,NP_CNAT,NP_CNAT_GUID,NP_CNNBT,NP_CNNBT_GUID,
     $ NP_CYBT,NP_CYBT_GUID,NP_HWNDTBL,NP_P1T,NP_P3T,NP_RAMPTB,
     $ NP_ROLL,NP_SOLFLOW,NP_SOLFLOW_GUID,NP_SOLTRST,
     $NP_SOLTRST_GUID,NP_THRSTCUT,NP_RWNDTBL,NP_RW_GUID,NP_THTWNDTBL,
     $ NP_THTW_GUID,NP_XCG,NP_XCG_GUID,NP_XMASSO_GUID,NP_Y1T,
     $ NP_Y3T,NP_YCG,NP_YCG_GUID,NP_ZCG,NP_ZCG_GUID

      DIMENSION XTF(6), QF(6), A13EQ(3,3), A13EQT(3,3), Q0S(6),
     $          X013(6), UT0(3), UDT0(3)
      DIMENSION A(80),GX(3),TAU(20),TCG(20),TFG(20),TGG(20),VEX(20)
      DOUBLE PRECISION ACC(3), XCG_GUID(36), YCG_GUID(36),
     $                  ZCG_GUID(36), EAST(3), NORTH(3), HT_GUID(156),
     $                  RW_GUID(156), THTW_GUID(156), RR(3),
     $                  ALT_BASE_GUID(100), QREF_GUID(100),
     $                  CFBST_GUID(100), DIAM(20), POS(3), VEL(3),
     $                  RENG(20,3), FA(3), NSOL, MAA(3), MASS_GUID(36)

      DOUBLE PRECISION PR(15), WIND(3), VRX(3), FENG(10),
     $                  VREL_EST(3), VRMAG
      DOUBLE PRECISION VB(3), CPP(3), AL(3,3), CPY(3)

      DIMENSION X(3),XD(3),XDDG(3),XF(3),FL(20),FJJ(20),S(20),Q(20),
     1          UU(20),XIT(3),XIDT(3),XIDDT(3),PHITM(3,3),XI(3),
     2          XID(3),XIDD(3),DXIDS(3),DXID(3),G(3,3),FK(3,3)
      DIMENSION ABCD(8),VEXSV(20),TFSV(20),IG2(20,2)
      DIMENSION A1(3,3),B1(3,3),TCSAVE(20)
      DOUBLE PRECISION TIMES_OLD(6), Q0_OLD(6), DTIMES_OLD(6),
     $                  DQ0_OLD(6), A_TARG_TO_EQ(3,2)

      DATA ALPHA_MIN / 0.8 /
      LOGICAL GUID_CONVERGED, First_Time/.true./
      DOUBLE PRECISION LAMBDA(3), LAMBDA_13(3), VR(3), Q0_LVLH(3),
     $                  Q_LVLH_TO_I(4), NWND, MACH, E_NUM(8,8),
     $                  Q0_NOM(6), C1(6), DC_NOM(8),
     $                  Z_NUM(12,12), XG_NOM(12)
      DATA C1 / 3*1.D-4, 3*1.D-7 /, T_LAST / 1.D+30 /
      COMMON/LINEAR/LINEAR_SOLUTION_METHOD
      COMMON/NORM_UNITS/NORM_VARS,D_UNIT,T_UNIT
      DOUBLE PRECISION VINF(3),DX(3)
      COMMON/PLANETARY/VINF
      COMMON/MAXKOUNT/ITER
        COMMON/AA_FORPRINT/AATARG
      LOGICAL NORM_VARS
      COMMON /OPGUID_DATA/ ISP,THLIQ,NCOAST,NBURN,ALPHA_MIN,
     $          ITARG,MITR,G0,CEXV,AZ,PHIL,C3OA,DECOA,RAOA,GUID_OPTION
      COMMON /ORBIT_INFO/ DINCL,DNODE,THT,ARGPER,TIME,IFLAG,
     $          ICIRC,TGT_SET,DELTABURN,R0,V0,RT,VT,XMASS,TMODE
        COMMON /Q0SUB/ Q0_SUB(6),Q1_SUB(6)
        COMMON /CURRENT_STATE/ XCUR(6)

        NORM_VARS = .FALSE.
        SOFT_CONSTRAINTS = .FALSE.
        MODE = TMODE
        UK = XMU/1.D+9
        RADDEG=180./PI
        DEGRAD=PI/180.
        MEAN_TGT=0
        MEANTGT = MEAN_TGT
        IMODE = MODE
        TOLC = .00001

C THESE VALUES ARE FOR MODE = 12
         VINFMAG = SQRT(C3OA)
         VINF(1) = VINFMAG * COSD(DECOA) * COSD(RAOA)
         VINF(2) = VINFMAG * COSD(DECOA) * SIND(RAOA)
         VINF(3) = VINFMAG * SIND(DECOA)
      THT = THT*DEGRAD
      AZ = AZ*DEGRAD
      PHIL = PHIL*DEGRAD


      IF ( TGT_SET .NE. TGT_SET_OLD ) THEN
C ------------------------------------------------------------------
C  TARGETING HAS ISSUED A NEW SET OF TARGETS.  RECOMPUTE TARGET FUNCTIONS
C ------------------------------------------------------------------
         BETA=0.
         BMT=0.
        DO I=1,6
          CC(I)=0.
          DD(I)=0.
        END DO

        IF ( ITARG .EQ. 2 ) THEN
C   ---------------------------------------
C   need to construct a cartesian target vector
C   ---------------------------------------
           IF ( ICIRC .EQ. 1 ) THEN
C   --------------------------
C   target orbit is non-circular
C   --------------------------
            P=(RT**2/UK)*VT**2*COS(THT)**2 ! SEMI-PARAMETER
            TRUAN=ATAN2(P*TAN(THT),P-RT)
            BETA=ARGPER+TRUAN  ! ARGUMENT OF LATITUDE
            BMT=BETA-THT
          END IF

         CNOD = COS(DNODE)
         SNOD = SIN(DNODE)
         CBETA=COS(BETA)
         SBETA=SIN(BETA)
         CINC=COS(DINCL)
         SINC=SIN(DINCL)
         CBMT=COS(BMT)
         SBMT=SIN(BMT)

       if (mode .ne. 6 .and. mode .ne. 7 .and. mode .ne. 13
     $         .and. mode .ne. 14) then
            XT(1)=RT*CBETA
            XT(2)=RT*CINC*SBETA
            XT(3)=-RT*SINC*SBETA
            XT(4)=-VT*SBMT
            XT(5)=VT*CINC*CBMT
            XT(6)=-VT*SINC*CBMT
       end if
       END IF !(ITARG = 2)

        Do i=1,6
          XTF(i) = XT(i)
        End do
        IF ( IFLAG .NE. 0 ) THEN
           DT = TIMES(6) - TIME
           CALL COAST716 (XT,E,DT,XTF,DUMY,DUMY,DUMY)
        END IF
        DO I = 1,6
          XG(I) = XT(I)
        END DO
C   ----------------------------------------------------------
C   compute orbital element target functions, i.e., the DD( )
C   ----------------------------------------------------------
        CALL BOUNDF(DUMY,DUMY,1)
C   -------------------------------------------------------------------
C   compute a convergence tolerance for future use in convergence test
C   -------------------------------------------------------------------
        Converg_limit = 0.
        DO I=1,6
          CC(I)=DD(I)  ! CC = Vector of Target Boundary Conditions
          Converg_limit = Converg_limit + cc(i)**2
        END DO
        CONVERG_LIMIT = TOLC * TOLC * CONVERG_LIMIT
     END IF
     IF ( ABS(TIME - T_LAST) .GT. 20. .OR. TGT_SET .NE.
    $                                       TGT_SET_OLD) THEN
C   ---------------------------------------------------------------------------
```

```
C  EITHER AT THE BEGINNING OF A NEW TRAJECTORY OR NEW TARGETS HAVE JUST BEEN
C  ISSUED.  RECOMPUTE INITIAL COSTATE GUESSES
C  ------------------------------------------------------------------------

          VR(1) = X0(4)  + X0(2)*OMEGA
          VR(2) = X0(5)  - X0(1)*OMEGA
          VR(3) = X0(6)
          V0=VMAG(X0(4))
C         V0 = VMAG(VR)
          VF=VMAG(XTF(4))
          RMAG = VMAG(X0)
          DO I=1,3
            QF(I)=XTF(I+3)/VF
            Q0(I) =  X0(I+3)/V0
            IF ( TIME .LT. -10. ) THEN
               Q0(I) = X0(I) / RMAG
            END IF
          End do
          DT=TIMES(6)-T0
          DO I=1,3
            Q0(I+3)=.001*(QF(I)-Q0(I))/DT !___end costate guess computation
          End do

          GUID_CONVERGED = .FALSE.
          MAX_IT = MITR
        END IF

         T_NAV = T0

c        PRINT 111, TT,T0
c        PRINT 112, ((VEH(I,J),J=1,7),I=1,10)
c        PRINT 113, X0
c        PRINT 114, XT
c        PRINT 115, Q0
c        PRINT 116, TIMES
c        PRINT 117, CC
C  ------------------------------------------------------------------
C  iteratively compute corrections to the costate and switching times
C  until the convergence criterion is met
C  ------------------------------------------------------------------
       ITERATIVE_MODE = 1
       DCM = 1.D+30
       IF ( (TIMES(6) - T0) .GT. 2. ) THEN
       IF ( ITERATIVE_MODE .EQ. 1 ) THEN
       DO 300 I=1,MAX_IT
         If ( DCM .le. Converg_limit ) GO TO 310
C  ----------------------
C  COMPUTE JACOBIAN MATRIX
C  ----------------------
         IF ( I .EQ. 1 ) DT_PRED = DT_GUID  ! compute a guidance solution one guid c

C NON-FLIGHT CODE
           CALL QUAT_LVLH_TO_I(X0, X0(4), Q_LVLH_TO_I)
           CALL QUATFORM(Q_LVLH_TO_I, Q0, Q0_LVLH)
           PITCH_LVLH = DATAN2(-Q0_LVLH(3), Q0_LVLH(1))*57.3
           YAW_LVLH = DATAN2(Q0_LVLH(2), DSQRT(Q0_LVLH(1)**2 +
     $                          Q0_LVLH(3)**2))*57.3
c           WRITE(30,*) ' LVLH PITCH ', PITCH_LVLH, ' LVLH YAW ', YAW_LVLH
C         write(76,*)sngl(t0),sngl(pitch_lvlh),sngl(yaw_lvlh)

C        ENSURE THAT THE OBLATE FACTOR IS ZERO IN GUIDE
         OBLATE = 0.
         NOP = 1
         DT_PRED = 0.
C
         CALL GUIDE(DT_PRED)
        DCM = 0.
        DO IL = 1,6
          DCM = DCM + DC(IL)**2
        END DO
        DCM_PAST = DCM
C       PRINT 349, iter
C       Print 355, (xG(j),j=1,12),cc,dd
C       Print 351, (DC(j),j=1,6)

C  ----------------
C  Convergence Test
c  ----------------

        If ( DCM .le. Converg_limit ) GO TO 310
        Iter = i
        IF ( GUID_CONVERGED ) THEN
           DO J = 1, 6
             TIMES(J) = TIMES(J) + DTIMES(J)/1.
             Q0(J) = Q0(J) + DQ0(J)/1.
           END DO
        ELSE
           ALPHA = 1.
           DO K = 1,6
              Q0_OLD(K) = Q0(K)
              TIMES_OLD(K) = TIMES(K)
              DQ0_OLD(K) = DQ0(K)/1.
              DTIMES_OLD(K) = DTIMES(K)/1.
           END DO
           DO WHILE ( (DCM .GE. DCM_PAST) .AND. (ALPHA .GT.
     $                                          ALPHA_MIN) )
             DO J=1,6
               TIMES(J) = TIMES_OLD(J) + ALPHA * DTIMES_OLD(J)
               Q0(J) = Q0_OLD(J) + ALPHA * DQ0_OLD(J)
             END DO
C NON-FLIGHT CODE
               CALL QUAT_LVLH_TO_I(X0, X0(4), Q_LVLH_TO_I)
               CALL QUATFORM(Q_LVLH_TO_I, Q0, Q0_LVLH)
               PITCH_LVLH = DATAN2(-Q0_LVLH(3), Q0_LVLH(1))*57.3
               YAW_LVLH = DATAN2(Q0_LVLH(2), DSQRT(Q0_LVLH(1)**2
     $                                 + Q0_LVLH(3)**2))*57.3
c              WRITE(30,*) ' LVLH PITCH ', PITCH_LVLH, ' LVLH YAW ', YAW_LVLH

             IF ( ALPHA / 2. .LE. ALPHA_MIN ) GO TO 300
             NOP = 0
             CALL GUIDE(0.D0)
             DCM = 0.
             DO IL = 1,6
               DCM = DCM + DC(IL)**2
             END DO
             ALPHA = ALPHA / 2.D0
           END DO
           DCM_PAST = DCM
         END IF
  300 CONTINUE

  310 Continue
C  --------------
C  NON-FLIGHT CODE
C  --------------
      ELSE  ! compute partials numerically to test OP GUID formulation of
         NOP = 1   ! variational equations
         CALL GUIDE(0.D0)
         NOP = 0
         WRITE(75,*)' FOLLOWING Z MATRIX COMPUTED USING VARIATIONS '
         WRITE(75,101)((SNGL(Z(K,J)),J=1,9),K=1,12)
         WRITE(75,*)' FOLLOWING E MATRIX COMPUTED USING VARIATIONS '
      WRITE(75,101)((SNGL(E(K,J)),J=1,9),K=1,9)
101   FORMAT(9E13.6)
      DO I = 1, 9
         DC_NOM(I) = DC(I)
      END DO
      DO I = 1, 12
         XG_NOM(I) = XG(I)
      END DO
      DO I = 1, 6
         Q0_NOM(I) = Q0(I)
      END DO
      TF_NOM = TIMES(6)
      BETA_NOM = 0.
      DO I = 1, 6
        DO J = 1, 6
          Q0(J) = Q0_NOM(J)
        END DO
        Q0(I) = Q0_NOM(I) + C1(I)
        CALL GUIDE(0.D0)
        DO J = 1, 9
          E_NUM(J,I) = (DC(J) - DC_NOM(J)) / (C1(I))
        END DO
        DO J = 1, 12
           Z_NUM(J,I) = (XG(J) - XG_NOM(J)) / (C1(I))
        END DO
      END DO
C ---------------------------------------
C NOW COMPUTE PARTIALS WRT SWITCHING TIMES
C ---------------------------------------
      NSWITCHES = 1

      IF ( TIMES(3) .GT. TIME ) THEN
         T3NOM = TIMES(3)
         TIMES(3) = T3NOM + 1.
         CALL DCOPY(6,Q0_NOM, 1, Q0, 1)
         CALL GUIDE(0.D0)
         DO J = 1, 10
             E_NUM(J, 6+NSWITCHES) = (DC(J) - DC_NOM(J)) / (TIMES(3)
     $                                                     -T3NOM)
         END DO
         TIMES(3) = T3NOM
         NSWITCHES = NSWITCHES + 1
      END IF
      IF ( TIMES(4) .GT. TIME ) THEN
         T4NOM = TIMES(4)
         TIMES(4) = T4NOM + 1.
         DO J = 1, 6
            Q0(J) = Q0_NOM(J)
         END DO
         CALL GUIDE(0.D0)
         DO J = 1, 9
           E_NUM(J,6+NSWITCHES) = (DC(J) - DC_NOM(J)) / (TIMES(4)
     $                                              - T4NOM)
         END DO
         DO J = 1, 12
            Z_NUM(J,6+NSWITCHES) = (XG(J) - XG_NOM(J)) / (TIMES(4)
     $                                                  - T4NOM)
         END DO
         TIMES(4) = T4NOM
         NSWITCHES = NSWITCHES + 1
      END IF
      IF ( TIMES(5) .GT. TIME ) THEN
         T5NOM = TIMES(5)
         TIMES(5) = T5NOM + 1.
         DO J = 1, 6
            Q0(J) = Q0_NOM(J)
            END DO
            CALL GUIDE(0.D0)
            DO J = 1, 9
              E_NUM(J,6+NSWITCHES) = (DC(J) - DC_NOM(J)) / (TIMES(5)
     $                                                    - T5NOM)
            END DO
            DO J = 1, 12
               Z_NUM(J,6+NSWITCHES) = (XG(J) - XG_NOM(J)) / (TIMES(5)
     $                                                         - T5NOM)
            END DO
            TIMES(5) = T5NOM
            NSWITCHES = NSWITCHES + 1
         END IF

         TIMES(6) = TF_NOM + 1.
         DO J = 1, 6
            Q0(J) = Q0_NOM(J)
         END DO
         CALL GUIDE(0.D0)
         DO J = 1, 10
           E_NUM(J,6+NSWITCHES) = (DC(J) - DC_NOM(J)) / (TIMES(6)
     $                                                    - TF_NOM)
         END DO
         DO J = 1, 12
            Z_NUM(J,6+NSWITCHES) = (XG(J) - XG_NOM(J)) / (TIMES(6)
     $                                                 - TF_NOM)
         END DO
         TIMES(6) = TF_NOM

         WRITE(75,*)' Z MATRIX GENERATED USING FINITE DIFFERENCES:'
         WRITE(75,101)((SNGL(Z_NUM(K,J)),J=1,9),K=1,12)
         WRITE(75,*)' E MATRIX GENERATED USING FINITE DIFFERENCES:'
         WRITE(75,101)((SNGL(E_NUM(K,J)),J=1,9),K=1,9)
       END IF
      END IF
      IF ( DCM .le. Converg_limit ) THEN
         GUID_CONVERGED = .TRUE.
         MAX_IT = 1
      END IF
C  -----------------------------------------------------------------
C  construct the guidance direction using a command predicted one guidance
C  cycle ahead
C  -----------------------------------------------------------------
       IF ( SAVED_GUID_PARAMS ) THEN
C  ---------------------------------------------------------------
C  construct the guidance pointing vector using atmospheric formulation
C  ---------------------------------------------------------------
          VRU = VR_0(1) * Q0(1) + VR_0(2) * Q0(2) + VR_0(3) * Q0(3)
          SY = CSV2_0 * VRU
          LAMBDA(1) = CSV1_0 * Q0(1) + SY * VR_0(1)
          LAMBDA(2) = CSV1_0 * Q0(2) + SY * VR_0(2)
          LAMBDA(3) = CSV1_0 * Q0(3) + SY * VR_0(3)
       ELSE
C  -----------------------------------------------------------
C  construct the guidance pointing vector using vacuum formulation
C  -----------------------------------------------------------
          LAMBDA(1) = Q0(1)
          LAMBDA(2) = Q0(2)
          LAMBDA(3) = Q0(3)
       END IF

C NON-FLIGHT CODE
              CALL QUAT_LVLH_TO_I(X0, X0(4), Q_LVLH_TO_I)
              CALL QUATFORM(Q_LVLH_TO_I, LAMBDA, Q0_LVLH)
              PITCH_LVLH = DATAN2(-Q0_LVLH(3), Q0_LVLH(1))*57.3
              YAW_LVLH = DATAN2(Q0_LVLH(2), DSQRT(Q0_LVLH(1)**2
     $                                     + Q0_LVLH(3)**2))*57.3
c              WRITE(30,*) ' LVLH PITCH ', PITCH_LVLH, ' LVLH YAW ', YAW_LVLH
C            WRITE(76,*)SNGL(T0),SNGL(PITCH_LVLH),SNGL(YAW_LVLH)
C  ----------------------------------------------
C  construct commanded pitch and yaw angles and rates
C  ----------------------------------------------
C       CALL ATT_COMP(ITIME,TGG,TFG,DT_GUID,TCHIT,TI,CHITP,CHITY,
C      1             FK,CHIDP,CHIDY,IDIMEN,IBURNG,
C      2              CHIPCN,CHIYCN,XPM1,XYM1,XDLIMP,XDLIMY,LAMBDA_13,
C      $              DPITCH,DYAW)
C         WRITE(76,*)SNGL(T0),SNGL(CHIPCN),SNGL(CHIYCN)
C***THIS SECTION OF CODE INSERTED TO CORRECT THE COMMANDED STEERING
C***ANGLE FOR THE MOMENT BALANCE GIMBAL ANGLE
C***SO THAT THE COMMANDED THRUSTING DIRECTION WILL BE AS DESIRED.
C       IF ( IMOMENT .EQ. 1 ) THEN
C***ASSUME THAT THE MOST RECENT GIMBAL VALUE IS AVAILABLE (IN FLIGHT,
C***IT MIGHT BE THE AVERAGED VALUE OVER THE LAST GUIDANCE CYCLE).

C  MODIFY PITCH AND YAW ANGLES

C         PITCH = ATAN2(LAMBDA_13(1),LAMBDA_13(3))
C         YAW = ATAN2(LAMBDA_13(2), DSQRT(LAMBDA_13(1)**2+
C      $                                 LAMBDA_13(3)**2))
C         PITCH = PITCH - DELP
C         YAW = YAW - DELY

C         LAMBDA_13(1) = COS(YAW) * SIN(PITCH)
C         LAMBDA_13(2) = SIN(YAW)
C         LAMBDA_13(3) = COS(YAW) * COS(PITCH)
C       CALL ATT_COMP(ITIME,TGG,TFG,DT_GUID,TCHIT,TI,CHITP,CHITY,
C      1             FK,CHIDP,CHIDY,IDIMEN,IBURNG,
C      2              CHIPCN,CHIYCN,XPM1,XYM1,XDLIMP,XDLIMY,LAMBDA_13,
C      $              DPITCH,DYAW)
C       END IF
C*** HERE, HAVE DESIRED THRUST DIRECTION.  STEERING ANGLE HAS BEEN
C*** MODIFIED SO THAT THRUST WILL BE AS DESIRED.
C       IF ( IAEROOPT .EQ. 1 ) THEN
C         PR(1)=HEIGHT
C         IF ( IATMOS_GUID .EQ. 1 ) THEN
C           CALL PRA63(PR,ERROR)
C         END IF
C         CALL LINLUM(156,HEIGHT,HT_GUID(1),RW_GUID(1),RWND)
C         CALL LINLUM(156,HEIGHT,HT_GUID(1),THTW_GUID(1),THTWND)

C***NEED R,THETA CALCULATION****************
C******************************************

C         DO I = 1, 3
C           WIND(I)=NWND*NORTH(I)+EWND*EAST(I)
C           VRX(I)=VEL(I)-RR(I)-WIND(I)
C         END DO
C         VRMAG=VMAG(VRX)
C         RHO=PR(6)
C         VSOUND=PR(9)
C         MACH = VRMAG/VSOUND
C         PRESA=PR(2)*10000.
C         DYNPRS = 0.5*VRMAG*VRMAG*RHO

C***NOTE FM INCLUDES DRAG AND BASE FORCE, BUT ASSSUMES THEY
C***WILL BE CONSTANT IN THE FUTURE.
C       THRUST = FM*XMASSO
C       J=NENG
C       IF ( IEOUTSV .EQ. 1 ) J = J - 1
C       DO I = 1,NENG
C         IF(IEOUTSV.EQ.1.AND.I.EQ.IENG) THEN
C           FENG(I)=0.
```

```
C         ELSE
C           FENG(I)=THRUST/J
C         END IF
C       END DO

C***GET AERODYNAMIC ANGLES FOR MAX QALPHA AND QBETA TESTS:
C         DO I=1,3
C           VREL_EST(I)=VRX(I)+ACC(I)*DT_GUID
C         END DO
C         CALL AEROGUIDSUB(MACH,DYNPRS,VREL_EST,CHIPCN,CHIYCN,CHIR,
C      &  IAFLG,VB,FA,CPP,MAA,AL,ALPHAP,ALPHAY,IBURN,CPY,DIAM)
C         QA = ALPHAP*RADDEG*DYNPRS*0.2048/GO
C         QB = ALPHAY*RADDEG*DYNPRS*0.2048/GO
C         DELTAALP=0.
C         DELTABETA=0.
C         IF(DABS(QA).GT.QAMAXLIM) THEN
C           AMAX=QAMAXLIM/RADDEG/DYNPRS/0.2048*GO
C           IF(ALPHAP.LT.0.) AMAX=-AMAX
C           DELTAALP = ALPHAP-AMAX
C         END IF
C         IF(DABS(QB).GT.QBMAXLIM) THEN
C           BMAX=QBMAXLIM/RADDEG/DYNPRS/0.2048*GO
C           IF(ALPHAY.LT.0.) BMAX=-BMAX
C           DELTABETA=ALPHAY-BMAX
C         END IF
C***MODIFY PITCH BY AMOUNT TO GET DESIRED LOADS AND BY RATE LIMITED
C***AMOUNT
C         PITCH = ATAN2(LAMBDA_13(1),LAMBDA_13(3))
C         YAW = ATAN2(LAMBDA_13(2), DSQRT(LAMBDA_13(1)**2+
C      $                                    LAMBDA_13(3)**2))
C         PITCH = PITCH - DELTAALP + DPITCH
C         YAW = YAW - DELTABETA + DYAW
C         LAMBDA_13(1) = COS(YAW) * SIN(PITCH)
C         LAMBDA_13(2) = SIN(YAW)
C         LAMBDA_13(3) = COS(YAW) * COS(PITCH)

C       CALL ATT_COMP(ITIME,TGG,TFG,DT_GUID,TCHIT,TI,CHITP,CHITY,
C      1              FK,CHIDP,CHIDY,IDIMEN,IBURNG,
C      2               CHIPCN,CHIYCN,XPM1,XYM1,XDLIMP,XDLIMY,LAMBDA_13,
C      $               DPITCH,DYAW)
C       END IF

C       WRITE(76,*)SNGL(T0),SNGL(CHIPCN),SNGL(CHIYCN)

C       T0 = T_NAV
C       T_LAST = T_NAV

c       PRINT 349, Iter, 2. * ALPHA, DCM
c        PRINT*,' CURRENT TIME ', T0, ' T CUTOFF ', TIMES(6)
c        PRINT 351, (DC(J),J=1,6)
c        PRINT 353
c        PRINT 350, Q0
c        print 354, ut0, udt0
c        PRINT 352, TIMES
c        PRINT 361
c    96 FORMAT(1H1)
c   111 FORMAT(1H ,3HTT=,F10.4,5H, T0=,F10.4)
c   112 FORMAT(1H ,' VEH    =',1P7E15.6)
c   113 FORMAT(1H ,' X0(Eq) =',1P6E15.6)
c   114 FORMAT(1H ,' XT(Eq) =',1P6E15.6)
c   115 FORMAT(1H ,' Q0(Eq) =',1P6E15.6)
c   116 FORMAT(1H ,' TIMES  =',1P6E15.6)
c   117 FORMAT(1H ,' CC(Eq) =',1P6E15.6)
C   349 FORMAT(' ITER ',I5, ' ALPHA ', D12.3, ' DCM ', D12.3)
c   350 FORMAT(' Q0(Eq)    =',6E15.6)
c   354 format(' Q0(AP13) =',6e15.6)
```

```
C  351 FORMAT(' DC(Eq)    =',6E12.2)
C  355 format('  R='3e15.7,6x,' V='3e15.7,/,'  U='3e15.7,6x,'UD='3e15.7/
C    1        ' CC='6e15.7,/,' DD='6e15.7)
c  352 FORMAT(6H TIMES,6E15.6)
c  353 FORMAT(40X,' VALUES FOR NEXT ITERATION')
c  361 FORMAT(45X,10H**********,//)

        FIRST_TIME = .FALSE.
        XMASS = SM(5)
        DO J = 1, 6
          XCUR(J) = XG(J)
        END DO

      RETURN
      END
```

THE FOLLOWING DATA FILE IS ONE THAT WAS USED TO COMPUTE THE MINIMUM FUEL TRANSFER FROM LEO TO GEO USING DELTABURN LENGTHS OF 1200 SECONDS.

```
$G715P_C
        ARGPER=0.,
        ISP = 450.,
        G0 = .0098,    !KILOMETERS
        NBURN = 1,
        NCOAST = 1,
        TIME =0.,
        THLIQ = 2.646,  !KILONEWTONS
        T0 = 0.,    !INITIAL TIME
        XMASSINIT=270000.,
        THT=0.,
        DINCL=10.,
        DNODE=0.,
        AZ = 55.2506,
        PHIL = 28.6084,
        IFLAG=0,
        GUID_OPTION = 2, !2=OPGUID
        TIMES=0.,0.,0.,0.,0.,1200.,
        VEH(1,1)=270000.,
        VEH(1,2)=.60,
        VEH(1,3)=1.D30,
        VEH(1,4)=2.646,
        VEH(1,5)=0.,
        VEH(1,6)=0.,
        VEH(1,7)=30.,
        MITR=250,
        ALPHA_MIN = .8,
        SOFT_CONSTRAINTS=.FALSE.,
        TMODE=2,
        ITARG=2,
        ICIRC=1,
        TGT_SET=1,
        OBLATE = 0.,
        LINEAR_SOLUTION_METHOD=5,
        DELTABURN=1200.,
        CEXV = 4.41,
        RPER=6656.,
        RAPO=6656.,
        SMATARG = 42164.,
        C3OA=160.,
        DECOA=0.,
        RAOA=120.001,
 $END
```

```
      SUBROUTINE KEPSTATE(R,V,RMAG,VMAG,SMA,ECC,INC,APE,OGA,TAN)
C     THIS PROGRAM TRANSFORMS THE SIX KEPLERIAN ELEMENTS A,E,I,W,O,AND
C     F (true anomaly) IN KILOMETERS TO A STATE VECTOR OF POSITION AND
C     VELOCITY IN KILOMETERS

      IMPLICIT DOUBLE PRECISION (A-H, O-Z)
      DOUBLE PRECISION R(3),V(3), INC, MU
      PARAMETER(MU = 398601.2D0, PI = 3.141592654D0)

      DEGRAD = PI/180.
      RADDEG = 180./PI

      INC = INC*DEGRAD
      APE = APE*DEGRAD
      OGA = OGA*DEGRAD
      TAN = TAN*DEGRAD

      SLA = SMA*(1.0 - ECC**2)
      FACTOR = SLA / (1.D0+ECC*DCOS(TAN))
      SQMUP = DSQRT(MU/SLA)
      COSFE = DCOS(TAN) + ECC

      CWCO=DCOS(APE)*DCOS(OGA) - DCOS(INC)*DSIN(OGA)*DSIN(APE)
      SWCO=-DSIN(APE)*DCOS(OGA)-DCOS(INC)*DSIN(OGA)*DCOS(APE)
      R(1)=(DCOS(TAN)*CWCO+DSIN(TAN)*SWCO)*FACTOR

      CWSO=DCOS(APE)*DSIN(OGA)+DCOS(INC)*DCOS(OGA)*DSIN(APE)
      SWSO=-DSIN(APE)*DSIN(OGA)+DCOS(INC)*DCOS(OGA)*DCOS(APE)
      R(2)=(DCOS(TAN)*CWSO+DSIN(TAN)*SWSO)*FACTOR

      R(3)=(DCOS(TAN)*DSIN(INC)*DSIN(APE)+DSIN(TAN)*DSIN(INC)*DCOS(APE))
     & *FACTOR

      V(1)=SQMUP*(COSFE*SWCO-DSIN(TAN)*CWCO)
      V(2)=SQMUP*(COSFE*(-DSIN(APE)*DCOS(OGA)+DCOS(INC)*DCOS(OGA)*DCOS(APE))
     & -DSIN(TAN)*CWSO)
      V(3)=SQMUP*(COSFE*(DSIN(INC)*DCOS(APE)-DSIN(TAN)*DSIN(INC)*DSIN(APE)))

      WRITE(19,100) 'POSITION','VELOCITY'
      DO I = 1, 3
        WRITE(19,101) R(I),V(I)
      END DO

      RMAG = DSQRT(R(1)**2+R(2)**2+R(3)**2)
      VMAG = DSQRT(V(1)**2+V(2)**2+V(3)**2)

      WRITE(19,*) 'IN KILOMETERS, RMAG IS ', RMAG
      WRITE(19,*) 'IN KILOMETERS, VMAG IS ', VMAG

100   FORMAT (1H, A, 32X, A)
101   FORMAT(1H, F28.16,8X,F28.16)
      END
      PROGRAM OPTI
C     THIS IS THE CALLING PROGRAM FOR THE MINIMUM PROPELLANT PROGRAM.
      CALL INPUT
      CALL ITERN
      END
      SUBROUTINE INPUT
C***********************************************************************
C     INPUT CALCULATES THE INITIAL AND FINAL ORBIT IN TERMS OF THE
C     EQUINOCTIAL ELEMENTS USING THE INPUT OF CLASSICAL ELEMENTS.
C
C     MAIN VARIABLES: -ZCUR(6),Z0(6) IS THE CURRENT AND INITIAL STATE
C                     -COCUR(6),LAM(6) IS THE CURRENT AND INITIAL COSTATE
C                     -ZF IS THE DESIRED FINAL STATE
C                     -POWR,C,MRATE ARE POWER,EXHAUST VELOCITY AND MASS
C                     RATE; POWER = -MRATE/C**2.
C***********************************************************************

      PARAMETER (N = 6)
      IMPLICIT DOUBLE PRECISION (A-H, O-Z)
      DOUBLE PRECISION  INC0, INCF, M0, LAM(N), MRATE
      DIMENSION  Z0(N), ZCUR(N), ZF(N), CO0(N), COCUR(N), COF(N)

      COMMON /STATE/ ZCUR, COCUR
      COMMON /CONST/ POWR, C, AMU
      COMMON /MASSES/ ZM0, ZMCUR, ZMF
      COMMON /ZFINAL/ ZF
      COMMON /ECC_LONG/ FECC1, FECC2

      NAMELIST  /OP_DATA/ A0,E0,INC0,W0,OMEGA0,FECC1,M0,AF,EF,INCF,WF,
     &          OMEGAF,FECC2,LAM,AMU,MRATE,C,POWR

C     THESE ARE VALUES USED FOR COMPILATION OF THE PROGRAM.
      READ(98, OP_DATA)

C     CALCULATE THE INITIAL AND FINAL STATES
      Z0(1) = A0
      Z0(2) = E0 * DSIN(W0 + OMEGA0)
      Z0(3) = E0 * DCOS(W0 + OMEGA0)
      Z0(4) = DTAN(INC0 / 2.D0) * DSIN(OMEGA0)
      Z0(5) = DTAN(INC0 / 2.D0) * DCOS(OMEGA0)
      Z0(6) = M0
      ZF(1) = AF
      ZF(2) = EF * DSIN(WF + OMEGAF)
      ZF(3) = EF * DCOS(WF + OMEGAF)
      ZF(4) = DTAN(INCF / 2.D0) * DSIN(OMEGAF)
      ZF(5) = DTAN(INCF / 2.D0) * DCOS(OMEGAF)

      CALL DCOPY(N,LAM,1,COCUR,1)

C     INITIALIZE THE MASS AND ASSIGN DATA TO ALL VECTORS
      ZMCUR = M0
      DO 5 I = 1, N
  5    ZCUR(I) = Z0(I)
      RETURN
      END

      SUBROUTINE ITERN
C***********************************************************************
C     ITERN IS THE CALLING ROUTINE FOR A THE NUMERICAL INTEGRATOR WHICH WILL
C     FIND THE INITIAL COSTATE VECTOR WHICH SATISFIES THE TARGET CONDITIONS.
C     AT THIS TIME THE SUBROUTINE SECANT  AND CALLING ROUTINES (WRITTEN
C     BY GREG DUKEMAN) ARE USED TO ITERATE ON THE COSTATE.
C***********************************************************************

      IMPLICIT DOUBLE PRECISION (A-H, O-Z)
      PARAMETER (ATHRESH = 7500.D0, N = 6, ITMAX = 250, MAXCALLS = 5)
      DIMENSION  XM1(N), X0(N), F(N), ZCUR(N), COCUR(N), XVAL(N), FOLD(N)

      LOGICAL  AOKAY
      EXTERNAL  FUNCT, SECANT
      COMMON /STATE/ ZCUR, COCUR

      AOKAY = .TRUE.
      NWRITE = 36
      EPSI = .005D0
C     INPUT THE FIRST GUESSES OF THE COSTATE VARIABLES, XM1 AND X0.
C     XM1(6) REPRESENTS THE COSTATE MASS.

      DO 20 I = 1, N
        XM1(I) = COCUR(I)
        X0(I)=COCUR(I)+EPSI
 20   CONTINUE

      CALL SECANT (XM1, X0, EPSI, N, NWRITE,F, FUNCT)

      WRITE(NWRITE, 10) 'THE CURRENT STATE IS:',(ZCUR(I),I=1,N)
      WRITE(NWRITE, 10) 'THE CURRENT COSTATE IS: ',(COCUR(I),I=1,N)
      WRITE(NWRITE, 10) 'THE FINAL CONDITIONS DIFFERENCES IS: ',
     & (F(I),I=1,N)


C     DETERMINE IF THE SEMIMAJOR AXIS IS OUT OF RANGE
      IF (ZCUR(1) .GT. ATHRESH) AOKAY = .FALSE.
      IF (.NOT. AOKAY) THEN
        WRITE(NWRITE, *) 'TRY A NEW INITIAL PAIR OF GUESSES SINCE A
     +                 IS OUT OF RANGE.'
        RETURN
      ENDIF
 10   FORMAT(/A/6(F25.12/)/)
      RETURN
      END

    SUBROUTINE SECANT(XOLD, X, EPSI, N, NWRITE, F, FUNCT)
C
C  This subroutine is the n-dimensional extension of the
C  well-known secant method (which solves the nonlinear
C  scalar equation F(x) = 0).
C
C  ***************INPUTS*********************************
C  XOLD    -  old estimate of solution vector x
C  X       -  current estimate of solution vector x
C  EPSI    -  termination criterion
C  N       -  the dimension of the system (currently max of 100)
C  NWRITE  -  the logical unit number to send output to
C  FUNCT   -  the user-supplied routine used to evaluate the
C             function values corresponding to trial solution vectors
C
C  ************  Outputs  ******************************
C
C  X  -  the solution vector
C  F  -  the function vector at the solution X
C
C  *****************************************************
C
     INTEGER I, IERROR, K, N, NFEVAL, NITER, NTRYS, NTRYSMAX,
    $        NWRITE
     DOUBLE PRECISION J(100, 100), DIFFX(100), F(N), XOLD(N),
    $                 X(N), XTRY(100), FTRY(100), C(100),
    $                 FOLD(100), POLD, DOTN, RNORM, ALPHA, EPSI,
    $                 P, DELTAX, TEMP

     IF ( NWRITE .GT. 0 ) WRITE(NWRITE, 100)
```

```
      NITER = 0
      NFEVAL = 0
      NTRYSMAX = 100
      CALL FUNCT(X, F)
      NFEVAL = NFEVAL + 1
      P = DOTN(F, F, N)
      CALL FUNCT(XOLD, FOLD)
      NFEVAL = NFEVAL + 1
      POLD = DOTN(F, F, N)
      IF ( P .GT. POLD ) THEN
         DO I = 1, N
            TEMP = XOLD(I)
            XOLD(I) = X(I)
            X(I) = TEMP
            TEMP = FOLD(I)
            FOLD(I) = F(I)
            F(I) = TEMP
         END DO
      ELSE
         POLD = P
      END IF
      DO 2 I = 1, N
    2 DIFFX(I) = X(I) - XOLD(I)
    3 IF ( NWRITE .GT. 0 ) WRITE(NWRITE, 101)
     $          NITER, NFEVAL, ( F(I), X(I), I = 1, N )
      CALL VECTORASSIGNN(XTRY, X, N)
C
C  Compute the pseudo Jacobian matrix (dF/dX)
C
      IF ( N .GT. 1 ) THEN
         DO 4 I = 1, N
         XTRY(I) = XOLD(I)
         CALL FUNCT(XTRY, FTRY)
         NFEVAL = NFEVAL + 1
         DELTAX = DIFFX(I)
         XTRY(I) = X(I)
         DO 5 K = 1, N
         J(K,I) = ( F(K) - FTRY(K) ) / DELTAX
   5     CONTINUE
   4     CONTINUE
      ELSE
         J(1,1) = (F(1) - FOLD(1)) / DIFFX(1)
      END IF
C
C  Compute the n-vector J^-1 F ( = C )
C
      CALL GAUSS(J, F, C, N, 100, IERROR, RNORM)
      IF ( IERROR .EQ. 2 ) WRITE(NWRITE, *)' ERROR IN GAUSS ROUTINE'
C
C  Save the current design vector for the next iteration
C
      CALL VECTORASSIGNN(XOLD, X, N)
      CALL VECTORASSIGNN(FOLD, F, N)
C
C  Update the design vector and function vector
C
      ALPHA = .8D0
      NTRYS = 0
  17  DO 6 I = 1, N
   6  X(I) = XOLD(I) - ALPHA * C(I)
      CALL FUNCT(X, F)
      NFEVAL = NFEVAL + 1
C
C  COMPUTE THE PERFORMANCE INDEX P AND THEN SEE IF IT HAS DECREASED.
C  IF IT HAS INCREASED, THEN REDUCE THE STEP SIZE PARAMETER ALPHA
C  IN AN ATTEMPT TO OBTAIN A DECREASE IN P.
```

```
 C
       P = DOTN(F, F, N)
       IF ( P .GE. POLD ) THEN
          NTRYS = NTRYS + 1
          IF ( NTRYS .LT. NTRYSMAX ) THEN
             ALPHA = 0.5D0 * ALPHA
             GOTO 17
          ELSE
             WRITE(NWRITE, *)' NO CONVERGENCE'
             RETURN
          END IF
       ELSE
          POLD = P
          DO 11 I = 1, N
   11     DIFFX(I) = - ALPHA * C(I)
       END IF
       NITER = NITER + 1
C
C  Check termination criterion
C
       IF ((P .GT. EPSI) .AND. (DOTN(DIFFX, DIFFX, N) .GT. EPSI)) THEN
          GOTO 3
       ELSE
          IF ( NWRITE .GT. 0 ) THEN
             WRITE(NWRITE, *)' CONVERGED SOLUTION'
             WRITE(NWRITE, 101) NITER, NFEVAL, ( F(I), X(I),
     $                                                  I = 1, N )
          END IF
          RETURN
       END IF
C
C  Format statements
C
  100 FORMAT(1H1,4X,12H# ITERATIONS,4X,15H#
     $ F EVALUATIONS,4X,20HBEST FUNCTION VALUES,4X,16HDESIGN
     $VARIABLES///)
  101 FORMAT(/1H ,I10,I17,D29.10,D23.10/(1H ,D56.10,D23.10))
       END

         SUBROUTINE FUNCT(XVAL, FVAL)
C***************************************************************************
C        FUNCT EVALUATES THE SYSTEM OF NONLINEAR EQUATIONS FOR A GIVEN
C        SET OF INITIAL COSTATE VALUES IN ORDER TO SOLVE THE SYSTEM F(X)
C        = 0 BY COMPUTING TRAJECTORIES  AND EVALUATING THE FUNCTION F(X) WHICH
C        IS COMPOSED OF THE FINAL CONDITIONS--IN THIS CASE, TARGET SEMIMAJOR
C        AXIS, TARGET ECCENTRICITY AND TRANSVERSALITY CONDITION MASS COSTATE.
C***************************************************************************

         IMPLICIT DOUBLE PRECISION (A-H, O-Z)
         PARAMETER  (N=6, NWRITE=36)
         DIMENSION  XVAL(N), FVAL(N), ZCUR(N), COCUR(N),
     +   AMX(5,3), AMXP(5,3,5), Z12(2*N), DERZ12(2*N), ZF(N), TSTEP(3)
         LOGICAL  AVG,CHECK
         COMMON /STATE/ ZCUR, COCUR
         COMMON /MASSES/ ZM0, ZMCUR, ZMF
         COMMON /COMASS/ COM0, COMCUR, COMF
         COMMON /AVG_CHECK/ CHECK
         COMMON /CONST/ POWR, C, AMU
         COMMON /DERIV_RK4/ DERZ12, Z12
         COMMON /ZFINAL/ ZF
         COMMON /ECC_LONG/ FECC1, FECC2
         EXTERNAL  RKFCT, RK4

         TWOPI = 8.D0*DATAN(1.D0)
         CHECK = .FALSE.
         DO 5 I = 1, N
  5       COCUR(I) = XVAL(I)
        COMCUR = XVAL(N)
C
        DO 6 I = 1, N
          Z12(I) = ZCUR(I)
          Z12(I+N) = COCUR(I)
  6     CONTINUE
        NDIM = 12
        TSTEP(1) = 0.D0
        TSTEP(2) = .005D0
        TSTEP(3) = .005D0

        T = TSTEP(1)
        TF = TSTEP(2)
        DT = TSTEP(3)

C***INTEGRATE STATE AND COSTATE ACROSS A STEP***
        DO WHILE (T .LT. TF)
        CALL RK4 (T,DT,TF,Z12,NDIM,RKFCT)
        WRITE(NWRITE,*) ' FOR TIME T = ',T
        WRITE(NWRITE,10) ' THE CURRENT STATE IS:',(Z12(I),I=1,N)
        WRITE(NWRITE,10) ' THE CURRENT COSTATE IS:',(Z12(I),I=7,2*N)
        END DO

        DO 7  I = 1, N
          ZCUR(I) = Z12(I)
          COCUR(I) = Z12(N+I)
 7      CONTINUE
        COMCUR = COCUR(N)

C CHECK FINAL CONDITIONS
        FVAL(1) = DABS(ZCUR(1) - ZF(1))
        FVAL(2) = DSQRT(ZCUR(2)**2+ZCUR(3)**2)
        FVAL(3) = DABS(COMCUR - 1.D0)
        FVAL(4)=0.D0
        FVAL(5)=0.D0
        FVAL(6)=0.D0

        FNORM = DOTN(FVAL,FVAL,N)
        WRITE(NWRITE,*) 'THE VALUE OF FNORM IS: ',FNORM,'AT T =',T

  10    FORMAT(/A/6(F25.12/)/)
        RETURN
        END

      SUBROUTINE RK4(T, DT, TF, X, NX, DESUB)
C
C  This is the classical fourth-order Runge Kutta numerical integration
C  method.
C
C             Variable definitions . . . . .
C
C  T  (scalar) The current value of the independent variable
C  DT (scalar) The integration step size
C  TF (scalar) The final point in the independent variable to integrate to
C  X  (vector) The vector of states or dependent values
C  NX (integer) The number of first-order differential equations
C               to integrate
C  DESUB        The differential equations subroutine
C
      DOUBLE PRECISION X(30),XP(30),F1(30),F2(30),F3(30),F4(30), T,
     $                 DT, TP, TF
      INTEGER I, NX

      CALL DESUB(T,X,F1)
      IF(ABS(DT) .GT. ABS(TF-T))DT=TF-T
      DO 10 I=1,NX
   10 XP(I)=X(I)+DT*F1(I)/2.D0
      TP=T+DT/2.D0
      CALL DESUB(TP,XP,F2)
      DO 20 I=1,NX
   20 XP(I)=X(I)+DT*F2(I)/2.D0
      CALL DESUB(TP,XP,F3)
      DO 30 I=1,NX
   30 XP(I)=X(I)+DT*F3(I)
      TP=T+DT
      CALL DESUB(TP,XP,F4)
      DO 40 I=1,NX
   40 X(I)=X(I)+DT*(F1(I)/6.D0+F2(I)/3.D0+F3(I)/3.D0+F4(I)/6.D0)
      T=T+DT

      RETURN
      END
        SUBROUTINE RKFCT(T,Z12,DERZ12)
C       RKFCT IS CALLED BY RK4--RUNGE-KUTTA INTEGRATOR--TO COMPUTE THE
C       AVERAGED DERIVATIVES TO BE INTEGRATED.

        PARAMETER(N = 6)
        IMPLICIT DOUBLE PRECISION (A-H, O-Z)
        LOGICAL  AVG, CHECK
        DIMENSION  Z12(12), DERZ12(12), ZCUR(N), COCUR(N),ZSUM(12),
     +  AMX(5,3), AMXP(5,3,5), DZ(N), ZDOTT(N), COZDOTT(N), G(12), H(12)
        COMMON /STATE/ ZCUR, COCUR
        COMMON /VEC12/ ZVEC, COVEC
        COMMON /MASSES/ ZM0, ZMCUR, ZMF
        COMMON /COMASS/ COM0, COMCUR, COMF
        COMMON /MPROD5/ DZ, PVLAM
        COMMON /AVG_CHECK/CHECK
        COMMON /AVERAGE/ AVG
        COMMON /CONST/ POWR, C, AMU
        COMMON /DERIV/ ZDOTT, ZMDOT, COZDOTT, COMDOT
        COMMON /ORBIT2/ X1,Y1,RA,PZ20,PZ26,PZ29,PZ35
        COMMON /ECC_LONG/ FECC1, FECC2
        EXTERNAL  EVALMP, PRIMER, AVGTST, INTEG, QTRAP, FCT

        TWOPI=8.D0*DATAN(1.D0)
        IFLAG = 2

C       CALL EVALMP(ZCUR,FECC1,AMU,AMX,AMXP,IFLAG)
C       CALL PRIMER(AMX,AMXP)
        AVG = .TRUE.
C       DO 3 I = 1, N
C         ZDOTT(I)=0.D0
C         COZDOTT(I)=0.D0
C         ZSUM(I)=0.D0
C         ZSUM(N+I)=0.D0
C 3     CONTINUE

C***THIS CHECK IS BASED ON THE NUMBER OF CALLS IN RK4
c       IF (CHECK) THEN
c         CALL AVGTST(AVG)
c       ENDIF

        IF (AVG) THEN
          CALL SWCHPTS
          CALL INTEG
        ELSE
C         PRINT*,'NOT AVERAGING NOW '
C***THIS CHANGE IS TO COMPUTE A 1 BURN ONLY 5/28/93
C         CALL QTRAP(FCT,0.D0,TWOPI,ZSUM,2*N,M)
          CALL QUAD(0.D0,TWOPI,FCT,ZSUM,Z12,G,H,2*N)
          DO 5 I = 1, N
```

```
            ZDOTT(I) = ZDOTT(I) + ZSUM(I)
            COZDOTT(I) = COZDOTT(I) + ZSUM(N+I)
  5     CONTINUE
        ENDIF

        DO 10 I = 1, N
          DERZ12(I) = ZDOTT(I)
          DERZ12(I+6) = COZDOTT(I)
  10    CONTINUE
        IF (ABS(ZDOTT(1)) .GT. 200.) AVG =.FALSE.

 2      RETURN
        END
        SUBROUTINE AVGTST (AVG)
 C      EVALUATE HOW FAST THE EQUINOCTIAL ELEMENTS ARE CHANGING

        IMPLICIT DOUBLE PRECISION (A-H,O-Z)
        DIMENSION  ZDOT(5), DZ(5)
        LOGICAL  AVG
        COMMON /MPROD5/ DZ, PVLAM
        COMMON /CONST/ POWR, C, AMU
        COMMON /MASSES/ ZM0, ZMCUR, ZMF
 C CALCULATE PARAMETER FOR STOPPING
        PARAMETER (THRESH = 100.D0)
        PARAMETER (IR=5,IC=3)

        AVG = .TRUE.
        DO 5 I = 1, 5
        ZDOT(I) = ((2.D0 * POWR) / ZMCUR * C) * DZ(I)
  5     CONTINUE
        IF (ZDOT(1) .GT. THRESH) THEN
          AVG = .FALSE.
 C        PRINT*, 'THE ORBITAL ELEMENTS ARE CHANGING TOO RAPIDLY TO AVERAGE.'
        ENDIF
        RETURN
        END
        SUBROUTINE SWCHPTS
 C PURPOSE: FIND THE ZEROES OF THE SWITCH FUNCTION SIGMA(F)
        PARAMETER(N = 6)
        IMPLICIT DOUBLE PRECISION (A-H, O-Z)
        DOUBLE PRECISION MESHPT(100),LOWER,MDPT,NP1,NP2,INCR
        INTEGER  PTON,FKOUNT
        DIMENSION LHS(100),FNP(100),ROOTS(100),ZCUR(N),LCUR(N)
        LOGICAL  STARTPOS,ENDPOS
        COMMON /STATE/ ZCUR, LCUR
        COMMON /CONST/ POWR, C, AMU
        COMMON /GREEKS/ ALPHA,BETA,DELTA,GAMMA
        COMMON /ENDDATA/ NP1, NP2, STARTPOS, ENDPOS
        COMMON /SWITCH/ROOTS,NROOT, FNP, FKOUNT, NKOUNT
        EXTERNAL SIGMAF, MYSIGN, DZBREN

        TWOPI = 8.D0*DATAN(1.D0)
        NP1 = 0.D0
        NP2 = TWOPI
        MAXFN = 100
        ERRABS = 1.D-3
        ERRREL = 1.D-4

        ALPHA = ZCUR(3) * LCUR(2) - ZCUR(2) * LCUR(3)
        BETA = (1.D0 + DSQRT(1.D0 - ZCUR(2)**2 - ZCUR(3)**2))**(-1)
        DELTA = DSQRT(AMU * ZCUR(1) * (1.D0 - ZCUR(2)**2 - ZCUR(3)**2))
        GAMMA = (0.5D0) * (1.D0 + ZCUR(4)**2 + ZCUR(5)**2)

        STARTPOS = .FALSE.
        ENDPOS = .FALSE.
        NROOT = 0
```

```
      NTOP = 0
      PTON = 0
      FKOUNT = 0
      NKOUNT = 61
      INCR = (NP2 - NP1) / NKOUNT
      MESHPT(1) = NP1

      LHS(1) = MYSIGN(SIGMAF(MESHPT(1)))
      DO 10 I = 2, NKOUNT
        MESHPT(I) = MESHPT(I-1) + INCR
        LHS(I) = MYSIGN(SIGMAF(MESHPT(I)))
10    CONTINUE

      DO 20 I = 1, NKOUNT - 1
        IF (LHS(I) .EQ. LHS(I+1)) THEN
          IF (LHS(I) .EQ. 0) THEN
            NROOT = NROOT + 1
            ROOTS(NROOT) = MESHPT(I)
          ENDIF
C  "MAY NEED TO ITERATE OR SAMPLE FOR EXACT OR MISSED ROOTS"
        ELSE
          LOWER = MESHPT(I)
          UPPER = MESHPT(I+1)
          CALL DZBREN(SIGMAF, ERRABS, ERRREL, LOWER, UPPER, MAXFN)
          NROOT = NROOT + 1
          ROOTS(NROOT) = UPPER

          IF (LHS(I) .LT. LHS(I+1)) THEN
            NTOP = NTOP + 1
            FNP(NTOP) = ROOTS(NROOT)
            FKOUNT = FKOUNT + 1
          ELSE
            PTON = PTON + 1
            FNP(PTON) = ROOTS(NROOT)
          ENDIF
        ENDIF
20    CONTINUE

      IF (NROOT .EQ. 0) ROOTS(1) = TWOPI

      IF (LHS(1) .GT. 0) STARTPOS = .TRUE.
      MDPT = (ROOTS(NROOT) + NP2) / 2.D0
      CHECKSIGN = MYSIGN(SIGMAF(MDPT))
      IF (CHECKSIGN .GT. 0) ENDPOS = .TRUE.
      RETURN
      END

      SUBROUTINE INTEG
C  PURPOSE: PREPARE INTEGRANDS FOR QUADRATURE AND THEN CALL QUADRATURE
C  ROUTINE, EITHER QUAD4, QUAD8, QUAD16 OR QUAD32

      IMPLICIT DOUBLE PRECISION (A-H, O-Z)
      DOUBLE PRECISION NP1, NP2, N
      PARAMETER(NN = 6)
      DIMENSION ZDOTT(NN),COZDOTT(NN),DERZ12(12),Z12(12),ZSUM(12),
     &  ROOTS(100),FNP(100),G(12),H(12)
      INTEGER FKOUNT
      LOGICAL STARTPOS,ENDPOS,THRUST
      COMMON /STATE/ ZCUR, COCUR
      COMMON /MPROD5/ DZ, PVLAM
      COMMON /AVERAGE/ AVG
      COMMON /MASSES/ ZM0, ZMCUR, ZMF
      COMMON /DERIV/ ZDOTT, ZMDOT, COZDOTT, COMDOT
      COMMON /DERIV_RK4/ DERZ12, Z12
      COMMON /ENDDATA/ NP1,NP2,STARTPOS,ENDPOS
      COMMON /SWITCH/ ROOTS, NROOT, FNP,FKOUNT, NKOUNT
      EXTERNAL QUAD, FCT,QTRAP

      IF (NROOT .EQ. 0) THRUST = .FALSE.
      IF (NROOT .EQ. 0) STARTPOS = .TRUE.
      HAVG = 0.D0
      COMDOT = 0.D0
      ZMDOT = 0.D0
      DO 3 I = 1, N
        ZDOTT(I) = 0.D0
        COZDOTT(I) = 0.D0
        ZSUM(I) = 0.D0
        ZSUM(NN+I) = 0.D0
 3    CONTINUE
      IF (STARTPOS) THEN
        CALL QUAD (NP1, ROOTS(1), FCT, ZSUM, Z12, G, H, 2*NN)
C       CALL QTRAP (FCT,NP1,ROOTS(1),ZSUM,2*NN,M)
      DO 20 I = 1, NN
        ZDOTT(I) = ZDOTT(I) + ZSUM(I)
        COZDOTT(I) = COZDOTT(I) + ZSUM(NN+I)
20    CONTINUE
      ENDIF

      IF (ENDPOS) THEN
        CALL QUAD (ROOTS(NROOT), NP2, FCT, ZSUM, Z12, G, H,2*NN)
C       CALL QTRAP (FCT,ROOTS(NROOT),NP2,ZSUM,2*NN,M)
      DO 30 I = 1, NN
        ZDOTT(I) = ZDOTT(I) + ZSUM(I)
        COZDOTT(I) = COZDOTT(I) + ZSUM(NN+I)
30    CONTINUE
      ENDIF
      IF (THRUST) THEN
        DO 40 I = 1, FKOUNT
        CALL QUAD (ROOTS(2*I), ROOTS(2*I+1), FCT, ZSUM, Z12, G, H, 2*NN)
C       CALL QTRAP (FCT,ROOTS(2*I),ROOTS(2*I+1),ZSUM,2*NN,M)
      DO 45 J = 1, NN
        ZDOTT(J) = ZDOTT(J) + ZSUM(J)
        COZDOTT(J) = COZDOTT(J) + ZSUM(NN+J)
45    CONTINUE
40    CONTINUE
      ELSE
      DO 50 I = 1, FKOUNT
      CALL QUAD (ROOTS(2*I-1), ROOTS(2*I), FCT, ZSUM, Z12, G, H, 2*NN)
C     CALL  QTRAP (FCT,ROOTS(2*I-1),ROOTS(2*I),ZSUM,2*NN,M)
      DO 55 J = 1, NN
        ZDOTT(J) = ZDOTT(J) + ZSUM(J)
        COZDOTT(J) = COZDOTT(J) + ZSUM(NN+J)
55    CONTINUE
50    CONTINUE
      ENDIF
      ZMDOT=ZDOTT(NN)
      COMDOT=COZDOTT(NN)
      RETURN
      END

      DOUBLE PRECISION FUNCTION SIGMAF(F)
C  PURPOSE: TO COMPUTE THE EXPRESSION OF THE FUNCTION LAMBDA
      IMPLICIT DOUBLE PRECISION (A-H, O-Z)
      DOUBLE PRECISION  LAMSQ, LAMOFF, LMCUR, LCUR, LVEC
      DIMENSION  ZCUR(6), LCUR(6), RVEC(3), VELVEC(3)
      INTEGER I, ILEN
      PARAMETER (ILEN=3)
      COMMON  /STATE/ ZCUR, LCUR
      COMMON  /CONST/ POWR, C, AMU
      COMMON /MASSES/ ZM0,ZMCUR,ZMF
      COMMON /COMASS/ COM0, LMCUR, COMF
      COMMON  /GREEKS/ ALPHA, BETA, DELTA, GAMMA
      EXTERNAL  DNRM2

      X1 = ZCUR(1)*((1.D0-(ZCUR(2)**2)*BETA)*DCOS(F)+ZCUR(2)*
     +     ZCUR(3)*BETA*DSIN(F)-ZCUR(3))
      Y1 = ZCUR(1)*((1.D0-(ZCUR(3)**2)*BETA)*DSIN(F)+ZCUR(2)*
     + ZCUR(3) * BETA*DCOS(F)-ZCUR(2))

      RVEC(1) = X1
      RVEC(2) = Y1
      RVEC(3) = 0.D0
      R = DNRM2(ILEN, RVEC, 1)

      XDOT = (ZCUR(2) * ZCUR(3) * BETA * DCOS(F) - (1.D0 - (ZCUR(2)**2)
     + * BETA) * DSIN(F)) * DSQRT(AMU * ZCUR(1)) / R

      YDOT = ((1.D0 - ZCUR(3)**2 * BETA) * DCOS(F) - ZCUR(2) *
     + ZCUR(3) * BETA * DSIN(F)) * DSQRT(AMU * ZCUR(1)) / R

      VELVEC(1) = XDOT
      VELVEC(2) = YDOT
      VELVEC(3) = 0.D0
      VEL = DNRM2(ILEN, VELVEC, 1)
      PROD1 = 2.D0*ZCUR(1)**2*XDOT*LCUR(1) + (Y1*XDOT-DELTA)*LCUR(2)
     +        - Y1*YDOT*LCUR(3)
      PROD2 = 2.D0*ZCUR(1)**2*YDOT*LCUR(1) - (X1*XDOT*LCUR(2))
     +        + (X1*YDOT+DELTA)*LCUR(3)
      PROD3 = (Y1*ZCUR(5) - X1*ZCUR(4))*ALPHA + GAMMA*(Y1*LCUR(4)+X1*LCUR(5))
      LAMSQ = (PROD1**2)/AMU**2 + (PROD2**2)/AMU**2 +(PROD3**2)/DELTA**2
      LAMOFF = DSQRT(LAMSQ)
      SIGMAF = LAMOFF - (ZMCUR * LMCUR / C)

      RETURN
      END

      DOUBLE PRECISION FUNCTION SZFF(F)
C  PURPOSE: CALCULATE THE FUNCTION S(Z,F) USED IN THE INTEGRATION IN
C       AVERAGING

      PARAMETER(N = 6)
      IMPLICIT DOUBLE PRECISION (A-H, O-Z)
      DOUBLE PRECISION  LCUR
      DIMENSION ZCUR(N), LCUR(N)
      COMMON /STATE/ ZCUR, LCUR

      FACTOR = (1.D0 - ZCUR(3)*DCOS(F) - ZCUR(2)*DSIN(F))
      SZFF = FACTOR / (8.D0 * DATAN(1.D0))
      RETURN
      END

      SUBROUTINE PRIMER(AMX,AMXP)
C     PRIMER CONSTRUCTS THE PRIMER VECTOR AND THE OPTIMAL THRUST
C     DIRECTION U^.
      IMPLICIT DOUBLE PRECISION (A-H, O-Z)
      PARAMETER (IM=5,IN=3,N=6)
      DIMENSION ZCUR(N),COCUR(N),COCUR5(5),UHAT(3),AMX(5,3),
     + AMXP(5,3,5),DZ(5),UVEC(3),UPROJ(3)
      COMMON /STATE/ ZCUR, COCUR
      COMMON /MPROD5/ DZ, PVLAM
      COMMON /HAT/ UHAT
      COMMON /ORBIT2/ X1,Y1,RA,PZ20,PZ26,PZ29,PZ35
      COMMON /ORBIT3/ X1DOT,Y1DOT

      EXTERNAL  DNRM2, DMURRV, DDOT

      DO 3 I = 1,IM
  3     COCUR5(I) = COCUR(I)
```

```
        CALL DMURRV(IM,IN,AMX,IM,IM,COCUR5,2,IN,UVEC)
        UVAL = (UVEC(1)*X1 + UVEC(2)*Y1)/(RA**2)
        UPROJ(1)=UVAL*X1/ZCUR(1)
        UPROJ(2)=UVAL*Y1/ZCUR(1)
        UPROJ(3)=0.
        DO 4 I=1, 3
  4     UVEC(I) = UVEC(I)-UPROJ(I)
C       UVEC(1)=X1DOT/ZCUR(1)
C       UVEC(2)=Y1DOT/ZCUR(1)
C       UVEC(3)=0.
        ULEN = DNRM2(IN, UVEC, 1)
        DO 5 I = 1, IN
  5       UHAT(I) = UVEC(I) / ULEN
        CALL DMURRV (IM,IN,AMX,IM,IN,UHAT,1,IM,DZ)
        PVLAM = DDOT(IM,COCUR5,1,DZ,1)
        RETURN
        END

      SUBROUTINE VECTORASSIGNN(X, Y)
C
C  This subroutine assigns the 3-element vector Y to the
C  3-element vector X
C
      DOUBLE PRECISION X(3), Y(3)

      X(1) = Y(1)
      X(2) = Y(2)
      X(3) = Y(3)

      RETURN
      END

      DOUBLE PRECISION FUNCTION DOTN(F,G,N)
      INTEGER  N
      DOUBLE PRECISION F(N),G(N)

      DOTN = 0.D0
      DO 10 I = 1,N
        DOTN = DOTN + F(I)*G(I)
   10 CONTINUE
      RETURN
      END

      SUBROUTINE GAUSS(A,B,X,N,MAINDM,IERROR,RNORM)
      INTEGER NM1, NP1, I, J, K, N, IP1, IERROR, IPIVOT, MAINDM
      DOUBLE PRECISION A(MAINDM, MAINDM), B(N), X(N),
     $                 RNORM
      REAL*16 AUG(100, 101), Q, RSQ, RESI, RMAG,
     $                 PIVOT, TEMP
      NM1=N-1
      NP1=N+1
C
C  SET UP THE AUGMENTED MATRIX FOR AX=B.
C
      DO 2 I=1,N
      DO 1 J=1,N
      AUG(I,J)=A(I,J)
    1 CONTINUE
      AUG(I,NP1)=B(I)
    2 CONTINUE
C
C  THE OUTER LOOP USES ELEMENTARY ROW OPERATIONS TO TRANSFORM
C  THE AUGMENTED MATRIX TO ECHELON FORM.
C
      DO 8 I=1,NM1
C
```

```
C  SEARCH FOR THE LARGEST ENTRY IN COLUMN I, ROWS I THROUGH N.
C  IPIVOT IS THE ROW INDEX OF THE LARGEST ENTRY.
C
      PIVOT=0.
      DO 3 J=I,N
      TEMP=ABS(AUG(J,I))
      IF(PIVOT .GE. TEMP)GOTO 3
      PIVOT=TEMP
      IPIVOT=J
    3 CONTINUE
      IF(PIVOT .EQ. 0.)GOTO 13
      IF(IPIVOT .EQ. I)GOTO 5
C
C  INTERCHANGE ROW I AND ROW IPIVOT.
C
      DO 4 K=I,NP1
      TEMP=AUG(I,K)
      AUG(I,K)=AUG(IPIVOT,K)
      AUG(IPIVOT,K)=TEMP
    4 CONTINUE
C
C  ZERO ENTRIES (I+1,I), (I+2,I),...,(N,I) IN THE AUGMENTED MATRIX.
C
    5 IP1=I+1
      DO 7 K=IP1,N
      Q=-AUG(K,I)/AUG(I,I)
      AUG(K,I)=0.
      DO 6 J=IP1,NP1
      AUG(K,J)=Q*AUG(I,J)+AUG(K,J)
    6 CONTINUE
    7 CONTINUE
    8 CONTINUE
      IF(AUG(N,N) .EQ. 0.)GOTO 13
C
C  BACKSOLVE TO OBTAIN A SOLUTION TO AX=B.
C
      X(N)=AUG(N,NP1)/AUG(N,N)
      DO 10 K=1,NM1
      Q=0.
      DO 9 J=1,K
      Q=Q+AUG(N-K,NP1-J)*X(NP1-J)
    9 CONTINUE
      X(N-K)=(AUG(N-K,NP1)-Q)/AUG(N-K,N-K)
   10 CONTINUE
C
C  CALCULATE THE NORM OF THE RESIDUAL VECTOR, B-AX.
C  SET IERROR=1 AND RETURN.
C
      RSQ=0.
      DO 12 I=1,N
      Q=0.
      DO 11 J=1,N
      Q=Q+A(I,J)*X(J)
   11 CONTINUE
      RESI=B(I)-Q
      RMAG=ABS(RESI)
      RSQ=RSQ+RMAG**2
   12 CONTINUE
      RNORM=SQRT(RSQ)
      IERROR=1
      RETURN
C
C  ABNORMAL RETURN --- REDUCTION TO ECHELON FORM PRODUCES A ZERO
C  ENTRY ON THE DIAGONAL.  THE MATRIX A MAY BE SINGULAR.
C
   13 IERROR=2
```

```
      RETURN
      END

        INTEGER FUNCTION MYSIGN(X)
        IMPLICIT DOUBLE PRECISION(A-H,O-Z)
C CALCULATE THE SIGN OF THE FUNCTION SIGMAF TO HELP DEFINE ENDPOINTS.

        MYSIGN = 0
        IF (X .GT. 0.D0) MYSIGN = 1
        IF (X .LT. 0.D0) MYSIGN = -1
        RETURN
        END

        SUBROUTINE FCT(F1,F2,Z12,H,G)
C       CALLED BY QUAD4 TO COMPUTE STATE AND COSTATE INTEGRANDS
        PARAMETER(N = 6)
        IMPLICIT DOUBLE PRECISION (A-H, O-Z)
        PARAMETER(IFLAG=1)
        DIMENSION  ZCUR(N),COCUR(N),AMX(5,3),HZ(5),
     +  AMXP(5,3,5), DZ(5), DHDLZ(5), DSDZ(5), DHDZ(5), UHMH(1,5),
     +  ANEWMX(5,3),H(12),G(12),UHAT(3),PROD(5),Z12(12)
        LOGICAL  AVG
        COMMON /STATE/ ZCUR, COCUR
        COMMON /CONST/ POWR, C, AMU
        COMMON /AVERAGE/ AVG
        COMMON /MASSES/ ZM0, ZMCUR, ZMF
        COMMON /COMASS/ COM0, COMCUR, COMF
        COMMON /MPROD5/ DZ, PVLAM
        COMMON /HAT/ UHAT
        COMMON /ORBIT2/ X1,Y1,RA,PZ20,PZ26,PZ29,PZ35
        EXTERNAL COMPHZ, EVALMP, PRIMER, SZFF, DDOT, DMURRV

        TWOPI = 8.D0 * DATAN(1.D0)
        DO 5  I = 1, 5
          DO 6 J = 1, 3
            ANEWMX(I,J) = 0.D0
  6     CONTINUE
  5     CONTINUE
        INTFLAG = 1
        F = F1
  8     CALL EVALMP (ZCUR, F, AMU, AMX, AMXP, IFLAG)
        CALL PRIMER (AMX,AMXP)
        DO 10 I = 1, 5
          UHMH(1,I) = DZ(I)
  10    CONTINUE
        DO 20 I = 1, 5
          IF (AVG) THEN
            DHDLZ(I) = ((2.D0 * POWR) / ZMCUR*C) * UHMH(1,I) * SZFF(F)
          ELSE
            DHDLZ(I) = ((2.D0 * POWR) / ZMCUR*C) * UHMH(1,I)
          ENDIF
  20    CONTINUE
        DO 30 I = 1, 5
          DO 35 J = 1, 3
            DO 40 K = 1, 5
              ANEWMX(K,J) = AMXP(K,J,I)
  40    CONTINUE
  35    CONTINUE

C       CHECK THE PREVIOUS CALCULATIONS USING ANOTHER IMSL ROUTINE

        CALL COMPHZ(AMXP,COCUR(5),UHAT,HZ)

C       HZ(I) = DBLINF(5,3,ANEWMX,5,COCUR,UHAT)

C       CALL DMURRV(5,3,ANEWMX,5,3,UHAT,1,5,PROD)
```

```
C        HZ(I) = DDOT(5,COCUR,1,PROD,1)
   30    CONTINUE

         DSDZ(1) = 0.D0
         DSDZ(2) = - DSIN(F) / TWOPI
         DSDZ(3) = - DCOS(F) /TWOPI
         DSDZ(4) = 0.D0
         DSDZ(5) = 0.D0


C        CALCULATE THE HAMILTONIAN OR THE AVERAGED HAMILTONIAN.

         IF (AVG) THEN
           HAM = ((2.D0 * POWR) / (ZMCUR * C)) * (PVLAM - ZMCUR*COMCUR/C)
     +     * SZFF(F)
         ELSE
           HAM = ((2.D0 * POWR) / (ZMCUR * C)) * (PVLAM - ZMCUR*COMCUR/C)
         ENDIF

         DO 50 I = 1, 5
           IF (AVG) THEN
             DHDZ(I) = -(((2.D0*POWR)/ZMCUR*C)*HZ(I)*SZFF(F) + HAM*DSDZ(I))
           ELSE
             DHDZ(I) = -(((2.D0*POWR)/ZMCUR*C)*HZ(I) + HAM*DSDZ(I))
           ENDIF
   50    CONTINUE


C        THIS PART CALCULATES THE PARTIAL DERIVATIVE DH/DLM.
         IF (AVG) THEN
           DHDLM = - ((2.D0 * POWR) / C**2) * SZFF(F)
         ELSE
           DHDLM = - ((2.D0 * POWR) / C**2)
         ENDIF

C        THIS PART CALCULATES THE PARTIAL DERIVATIVE DH/DM.

         IF (AVG) THEN
           DHDM =  (2.D0 * POWR) / (ZMCUR**2 * C) * PVLAM * SZFF(F)
         ELSE
           DHDM =  (2.D0 * POWR) / (ZMCUR**2 * C) * PVLAM
         ENDIF

C        IF (INTFLAG .EQ. 0) GOTO 65

         DO 60 I = 1, N-1
           H(I) = DHDLZ(I)
           H(N+I) = DHDZ(I)
   60    CONTINUE
           H(N) = DHDLM
           H(2*N) = DHDM
C        IF  (M .EQ. 1) THEN
C          F = F2
C          INTFLAG=0
C          GO TO 8
C        ENDIF
C   65   CONTINUE
C        DO 70 I = 1, N-1
C          G(I) = DHDLZ(I)
C          G(N+I) = -DHDZ(I)
C   70   CONTINUE
C        G(N) = DHDLM
C        G(2*N) = DHDM

         RETURN
         END
```

```
        SUBROUTINE COMPHZ(A,Y,X,HZ)
C THIS SUBROUTINE COMPUTES THE TENSOR NEEDED IN PARTIAL OF COSTATE
        IMPLICIT DOUBLE PRECISION (A-H,O-Z)
        DOUBLE PRECISION A(5,3,5),Y(5),X(3),HZ(5),KIJ(5,3,5)

        DO 10 I=1,5
        DO 20 J=1,3
        DO 30 K=1,5
          KIJ(I,J,K) = Y(K)*A(I,J,K)
  30    CONTINUE
  20    CONTINUE
  10    CONTINUE

        DO 32 I=1,5
  32    HZ(I) =0.D0

        DO 35 I=1,5
        DO 40 J=1,3
        DO 45 K=1,5
         HZ(I)=HZ(I)+KIJ(K,J,I)*X(J)
  45    CONTINUE
  40    CONTINUE
  35    CONTINUE

        RETURN
        END

      SUBROUTINE EVALMP(X, THETA, AMU, AM, PAM, IMFLAG)
C EVALMP/EVALMPC
C     THIS SUBROUTINE EVALUATES THE 5X3 MATRIX M AND THE
C     5X3X5 PARTIAL OF M WRT X
C
C     IF IMFLAG=1, BOTH M (AM) AND ITS PARTIAL (PAM) ARE EVALUATED
C     IF IMFLAG=2, ONLY M (AM) IS EVALUATED
C     IF IMFLAG=3, ONLY THE PARTIAL OF M (PAM) IS EVALUATED
C
C
      IMPLICIT DOUBLE PRECISION(A-H,O-Z), INTEGER (I-N)
      DIMENSION X(5), AM(5,3), PAM(5,3,5)
      COMMON / ORBIT2 / X1,Y1,RA,PZ20,PZ26,PZ29,PZ35
      COMMON /ORBIT3/ X1DOT, Y1DOT
C
C
      EN=DSQRT(AMU/X(1)**3)
        if ((x(2)**2 + x(3)**2) .ge. 1.d0) then
          print*, 'trouble in evalmp'
          return
        endif
      RHO= DSQRT(1.D0- X(2)**2- X(3)**2)
      BETA= 1.D0/(1.D0 +RHO)
      CT= DCOS(THETA)
      ST= DSIN(THETA)
      RA= 1.D0-X(3)*CT -X(2)*ST
      ZETA= X(3)*ST-X(2)*CT
      BETA3= BETA**3/(1.D0 -BETA)
      X1= X(1)*((1.D0 -X(2)**2*BETA)*CT +X(2)*X(3)*BETA*ST -X(3))
      Y1= X(1)*((1.D0 -X(3)**2*BETA)*ST +X(2)*X(3)*BETA*CT -X(2))
      X11=X1
      Y11=Y1
      X1DOT= -((1.D0 -X(2)**2*BETA)*ST -X(2)*X(3)*BETA*CT)*EN*X(1)/RA
      Y1DOT= ((1.D0 -X(3)**2*BETA)*CT -X(2)*X(3)*BETA*ST)*EN*X(1)/RA
      PZ1= X(1)*(ZETA*(BETA+X(2)**2*BETA3) -(X(2)*BETA -ST)*CT/RA)
      PZ2= -X(1)*(-ZETA*X(2)*X(3)*BETA3 +1.D0 +(ST -X(2)*BETA)*ST/RA)
      PZ3= X(1)*(-ZETA*X(2)*X(3)*BETA3-1.D0 +(X(3)*BETA -CT)*CT/RA)
      PZ4= X(1)*(-ZETA*(BETA +X(3)**2*BETA3) +(CT -X(3)*BETA)*ST/RA)
      IF (IMFLAG .EQ. 3) GO TO 10
C     IF DO NOT WANT TO EVALUATE PARTIAL OF M, BRANCH TO 10
      AM(1,1)= 2.D0*X1DOT/(EN**2*X(1))
      AM(1,2)= 2.D0*Y1DOT/(EN**2*X(1))
      AM(1,3)= 0.D0
      DUM= RHO/(EN*X(1)**2)
      AM(2,1)= DUM*(PZ2- X(2)*BETA*X1DOT/EN)
      AM(2,2)= DUM*(PZ4 -X(2)*BETA*Y1DOT/EN)
      AM(2,3)= DUM*(X(3)*(X(5)*Y1 -X(4)*X1))/RHO**2
      AM(3,1)= -DUM*(PZ1 +X(3)*BETA*X1DOT/EN)
      AM(3,2)= -DUM*(PZ3 +X(3)*BETA*Y1DOT/EN)
      AM(3,3)= -DUM*(X(2)*(X(5)*Y1 -X(4)*X1)/RHO**2)
      AM(4,1)= 0.D0
      AM(4,2)= 0.D0
      DUM= (1.D0 +X(4)**2 +X(5)**2)/(2.D0*EN*X(1)**2*RHO)
      AM(4,3)= DUM*Y1
      AM(5,1)= 0.D0
      AM(5,2)= 0.D0
      AM(5,3)= DUM*X1
      IF (IMFLAG .EQ. 2) RETURN
C     IF WE ONLY WISH TO EVALUATE M THEN PROGRAM RETURNS HERE
 10   CA= DSQRT(AMU/X(1))/RA
      PZ5= X(2)*BETA3
      PZ6= X(3)*BETA3
      PZ9= CA*ST/RA
      PZ10= CA*CT/RA
      PZ20= X(1)*(-2.D0*X(2)*BETA*CT +X(3)*BETA*ST +PZ5*ZETA*X(2))
      PZ26= X(1)*(X(2)*BETA*ST -1.D0 +PZ6*X(2)*ZETA)
      PZ29= X(1)*(X(3)*BETA*CT -1.D0 -PZ5*X(3)*ZETA)
      PZ35= X(1)*(-2.D0*X(3)*BETA*ST +X(2)*BETA*CT -PZ6*X(3)*ZETA)
      PZ11= -X1DOT/(2.D0*X(1))
      PZ12= -Y1DOT/(2.D0*X(1))
      DUM1= 1.D0 -RA
      PZ13= -CA*(-2.D0*X(2)*BETA*ST -X(3)*BETA*CT -PZ5*X(2)*DUM1)+PZ9
     1                         *X1DOT/CA
      PZ14= -CA*(-X(2)*BETA*CT -PZ6*X(2)*DUM1) +PZ10*X1DOT/CA
      PZ15= -CA*(X(3)*BETA*ST +PZ5*X(3)*DUM1) +PZ9*Y1DOT/CA
      PZ16= -CA*(2.D0*X(3)*BETA*CT +X(2)*BETA*ST +PZ6*DUM1*X(3))
     1          +PZ10*Y1DOT/CA
      DUM= BETA +X(2) *PZ5
      PZ17= 1.D0+ PZ5*X(2)*(3.D0/BETA +1.D0/(1.D0-BETA))
      PZ18= (2.D0 +PZ17)*PZ5
      PZ19= PZ17*PZ6
      DUM2= X(2)*BETA -ST
      PZ21= -X(1)*(CT*DUM -ZETA*PZ18 +CT*DUM/RA +CT*ST*DUM2/RA**2)
      PZ22= X(1)*(ST*DUM +ZETA*PZ19 -CT*X(2)*PZ6/RA-CT**2*DUM2/RA**2)
      PZ23= BETA3*(3.D0/BETA +1.D0/(1.D0 -BETA))
      PZ24= PZ23*PZ5
      PZ25=PZ23*PZ6
      PZ27= X(1)*(-CT*X(2)*X(3)*BETA3 +ZETA*X(3)*(BETA3 +X(2)*PZ24)
     1      +(ST*(BETA +X(2)*PZ5))/RA +ST**2*DUM2/RA**2)
      PZ28= X(1)*(ST*X(2)*X(3)*BETA3 +ZETA*X(2)*(BETA3 +X(3)*PZ25)
     1      +X(2)*ST*PZ6/RA +ST*CT*DUM2/RA**2)
      DUM2= X(3)*BETA-CT
      PZ30= X(1)*(CT*X(2)*X(3)*BETA3 -ZETA*X(3)*(BETA3 +X(2)*PZ24)
     1     +CT*X(3)*PZ5/RA +CT*ST*DUM2/RA**2)
      PZ31= X(1)*(-ST*X(2)*X(3)*BETA3 -ZETA*X(2)*(BETA3 +X(3)*PZ25)
     1       +CT*(BETA +X(3)*PZ6)/RA +CT**2*DUM2/RA**2)
      DUM= BETA +X(3)*PZ6
      PZ32= 1.D0 +PZ6*X(3)*(3.D0/BETA +1.D0/(1.D0 -BETA))
      PZ33= PZ32*PZ5
      PZ34= PZ32*PZ6 +2.D0*X(3)*BETA3
      PZ36= X(1)*(CT*DUM -ZETA*PZ33 -ST*X(3)*PZ5/RA -ST**2*DUM2/RA**2)
      PZ37= X(1)*(-ST*DUM -ZETA*PZ34 -ST*(BETA +X(3)*PZ6)/RA -ST*CT
     1          *DUM2/RA**2)
      DO 20 J=1,2
```

```
20    PAM(1,J,1)= 3.D0*AM(1,J)/(2.D0*X(1))
      DUM =2.D0*X(1)**2/AMU
      PAM(1,1,2)= PZ13*DUM
      PAM(1,1,3)= PZ14*DUM
      PAM(1,2,2)= PZ15*DUM
      PAM(1,2,3)=PZ16*DUM
      DUM= DSQRT(AMU*X(1))
      CB=RHO/DUM
      PZ38= -X(2)*CB/RHO**2
      PZ39= -X(3)*CB/RHO**2
      PAM(2,1,1)= AM(2,1)/(2.D0*X(1))
      PAM(2,1,2)= -CB*BETA*X1DOT/EN +PZ38*AM(2,1)/CB +CB*(PZ27
     1           -X(2)*BETA*PZ13/EN -X(2)*X1DOT*PZ5/EN)
      PAM(2,1,3)= PZ39*AM(2,1)/CB +CB*(PZ28 -PZ6*X(2)*X1DOT/EN
     1             -X(2)*BETA*PZ14/EN)
      PAM(2,2,1)= AM(2,2)/(2.D0*X(1))
      PAM(2,2,2)= PZ38*AM(2,2)/CB +CB*(PZ36 -BETA*Y1DOT/EN -X(2)
     1         *Y1DOT*PZ5/EN -X(2)*BETA*PZ15/EN)
      PAM(2,2,3)= PZ39*AM(2,2)/CB +CB*(PZ37 -X(2)*Y1DOT*PZ6/EN
     1            -X(2)*BETA*PZ16/EN)
      PAM(2,3,1)= AM(2,3)/(2.D0*X(1))
      DUM1= X(5)*Y1 -X(4)*X1
      PAM(2,3,2)= X(3)*(X(5)*PZ29 -X(4)*PZ20)/(RHO*DUM) +X(2)*X(3)
     1         *DUM1/(RHO**3*DUM)
      PAM(2,3,3)= DUM1/(RHO*DUM) +X(3)*(X(5)*PZ35 -X(4)*PZ26)/(RHO
     1        *DUM) +X(3)**2*DUM1/(RHO**3*DUM)
      PAM(2,3,4)= -X(3)*X1/(RHO*DUM)
      PAM(2,3,5)= X(3)*Y1/(RHO*DUM)
      PAM(3,1,1)= AM(3,1)/(2.D0*X(1))
      PAM(3,1,2)= PZ38*AM(3,1)/CB -CB*(PZ21 +X(3)*X1DOT*PZ5/EN
     1        +X(3)*BETA*PZ13/EN)
      PAM(3,1,3)= PZ39*AM(3,1)/CB -CB*(PZ22 +(BETA*X1DOT +X(3)
     1         *X1DOT*PZ6 +X(3)*BETA*PZ14)/EN)
      PAM(3,2,1)= AM(3,2)/(2.D0*X(1))
      PAM(3,2,2)= PZ38*AM(3,2)/CB -CB*(PZ30 +X(3)*(Y1DOT*PZ5
     1       +BETA*PZ15)/EN)
      PAM(3,2,3)= PZ39*AM(3,2)/CB -CB*(PZ31 +(BETA*Y1DOT +X(3)
     1        *Y1DOT*PZ6 +X(3)*BETA*PZ16)/EN)
      PAM(3,3,1)= AM(3,3)/(2.D0*X(1))
      PAM(3,3,2)= -DUM1/(RHO*DUM) -X(2)*(X(5)*PZ29 -X(4)*PZ20)/
     1        (RHO*DUM) -X(2)**2*DUM1/(RHO**3*DUM)
      PAM(3,3,3)= -X(2)*(X(5)*PZ35 -X(4)*PZ26)/(RHO*DUM) -X(2)*X(3)
     1         *DUM1/(RHO**3*DUM)
      PAM(3,3,4)= X(2)*X1/(RHO*DUM)
      PAM(3,3,5)= -X(2)*Y1/(RHO*DUM)
      Z5= (1.D0 +X(5)**2 +X(4)**2)/(2.D0*DUM*RHO)
      PZ40= -Z5/(2.D0*X(1))
      PZ41= X(2)*Z5/RHO**2
      PZ42= X(3)*Z5/RHO**2
      PZ43= X(4)/(DUM*RHO)
      PZ44= X(5)/(DUM*RHO)
      PAM(4,3,1)= AM(4,3)/(2.D0*X(1))
      PAM(4,3,2)= PZ41*Y1+Z5*PZ29
      PAM(4,3,3)= PZ42*Y1 +Z5*PZ35
      PAM(4,3,4)= PZ43*Y1
      PAM(4,3,5)= PZ44*Y1
      PAM(5,3,1)= AM(5,3)/(2.D0*X(1))
      PAM(5,3,2)= PZ41*X1 +Z5*PZ20
      PAM(5,3,3)= PZ42*X1 +Z5*PZ26
      PAM(5,3,4)= PZ43*X1
      PAM(5,3,5)= PZ44*X1
      DO 30 K=1,5
      PAM(1,3,K)=0.D0
      DO 30 I=4,5
      DO 30 J=1,2
30    PAM(I,J,K)=0.D0
```

```
      DO 40 I=1,3
      DO 40 J=1,2
      DO 40 K=4,5
 40   PAM(I,J,K)=0.D0
      RETURN
      END

      SUBROUTINE QUAD(XL,XU,FCT,Y,Z,G,H,N)
C
C QUAD
C
C     THIS IS A MODIFIED QUADRATURE PROGRAM FOR VECTOR VALUED FUNCTIONS.
C     COMPUTES INTEGRAL OF THE FUNCTION G (OR H) OVER X FROM XL TO XU.
C     THE RESULT IS Y.   ROUTINE USES A 4 POINT GAUSS QUADRATURE.
C
C
C
      IMPLICIT DOUBLE PRECISION(A-H,O-Z), INTEGER (I-N)
      DIMENSION Y(1),H(1),G(1),Z(1)
        EXTERNAL  FCT
C
      A= .5D0*(XU+XL)
      B= XU-XL
      C= .4305681557970262 9D0*B
      K= 1
      GO TO 50
 10   DO 20 I=1,N
 20      Y(I)= .1739274225687269 3D0*G(I)
      C= .1699905217924281 3D0*B
      K=2
      GO TO 50
 30   DO 40 I=1,N
 40      Y(I)= B*(Y(I) + .3260725774312730 7D0*G(I))
      RETURN
 50   CALL FCT(A-C,A+C,Z,H,G)
      DO 60 I=1,N
 60      G(I)=G(I) + H(I)
      GO TO (10,30), K
      END
      SUBROUTINE QTRAP(FUNC,A,B,S,N)
C       RETURNS S AS THE VECTOR INTEGRAL OF FUNCT OVER [A,B]
      IMPLICIT DOUBLE PRECISION(A-H,O-Z)
      PARAMETER(EPS=1.D-4, JMAX=20)
      DIMENSION S(12),SOLDS(12),G(12),H(12),SUM(12)
      EXTERNAL FUNC

      DO 10 I= 1,N
 10     SOLDS(I)=-1.D30

      DO 11 J = 1, JMAX
      M=J
      IF (M .EQ. 1) THEN
        CALL FUNC(A,B,H,G,M)
        DO 12 K=1,N
 12       S(K)=0.5D0*(B-A)*(G(K) + H(K))
          IT = 1
      ELSE
        ITNM=IT
        DEL=(B-A)/ITNM
        X=A+0.5D0*DEL
        DO 13 K=1,N
 13       SUM(K)=0.D0
      DO 20 K = 1, IT
        CALL FUNC(X,B,H,G,M)
        DO 25 I=1,N
 25     SUM(I)=SUM(I)+H(I)
```

```
          X=X+DEL
  20      CONTINUE
        DO 30 I=1,N
  30      S(I)=0.5D0*(S(I)+(B-A)*SUM(I)/ITNM)
        IT=2*IT
        ENDIF
        DO 35 I=1,N
        IF (DABS(S(I)-SOLDS(I)) .LT. EPS*(DABS(SOLDS(I)))) RETURN
        SOLDS(I)=S(I)
  35      CONTINUE
  11      CONTINUE
        IF (IT .GT. 50) PRINT*, 'TOO MANY STEPS'

        END
```

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16. Abstract

For many optimal transfer problems it is reasonable to expect that the minimum time solution is also the minimum fuel solution. However, if one allows the propulsion system to be turned off and back on, it is clear that these two solutions may differ. In general, high thrust transfers resemble the well known impulsive transfers where the burn arcs are of very short duration. The low and medium thrust transfers differ in that their thrust acceleration levels yield longer burn arcs and thus will require more revolutions. In this research, we considered two approaches for solving this problem; a powered flight guidance algorithm previously developed for higher thrust transfers was modified and an "averaging technique" was investigated.

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